Department of Economics- FEA/USP

Long-term stock returns in Brazil: volatile equity returns for U.S.-like investors

EURILTON ARAUJO
RICARDO D. BRITO
ANTONIO Z. SANVICENTE

WORKING PAPER SERIES Nº 2020-06

DEPARTMENT OF ECONOMICS, FEA-USP
WORKING PAPER Nº 2020-06

Long-term stock returns in Brazil: volatile equity returns for U.S.-like investors

Eurilton Araujo (BCB)

Ricardo D. Brito (FEA-USP, RicardoBrito@usp.br)

Antonio Z. Sanvicente (EESP/FGV)

Abstract:

This paper tells the history of Brazilian stock market returns since the creation of the Ibovespa (the main Brazilian stock market index). From 1968 to 2019, the arithmetic mean return of the Brazilian stock market is 21.3% per year. The equity premium is 20.1% per year, with a huge standard deviation of 67%. Surprisingly, such numbers are compatible with investors' risk aversions that accommodate the very different U.S. market evidence, reinforcing the belief that national investors are similar in nature. The equity premium has been higher in Brazil than in the U.S., but the much higher Brazilian volatility discourages heavier investments in stocks.

Keywords: Equity returns, Equity risk premium, Emerging market, Lifetime portfolio selection.

JEL Codes: E21, G10, G12

Long-term stock returns in Brazil: volatile equity returns for U.S.-like investors ℵ

Eurilton Araujo, Ricardo D. Brito and Antonio Z. Sanvicente

This version: March 30th, 2020

Abstract

This paper tells the history of Brazilian stock market returns since the creation of the Ibovespa (the main Brazilian stock market index). From 1968 to 2019, the arithmetic mean real return of the Brazilian stock market is 21.3% per year. The equity premium is 20.1% per year, with a huge annual standard deviation of 67%. Surprisingly, such numbers are compatible with investors' risk aversions that accommodate the very different U.S. market evidence, exposing that national investors are similar in nature. The equity premium has been higher in Brazil than in the U.S., but the much higher Brazilian volatility discourages heavier investments in stocks.

JEL: E21, G10, G12.

Keywords: Equity returns, Equity risk premium, Emerging market, Lifetime portfolio selection.

ℵ We appreciate the helpful comments of Rodrigo Bueno, Bruno Giovannetti, Bernardo Guimarães, Fang Liu, Marco Lyrio, Rafael Santos, seminar participants at Insper, Itaú-Unibanco Asset Management, UFABC, BCB and CVM/CeFi Workshops. Ítalo Franca provided excellent research assistance. Corresponding author: Ricardo Brito; E-mail: RicardoDOBrito@gmail.com; Postal address: Avenida Professor Luciano Gualberto, 908, São Paulo, SP 05508-010, Brazil. The views expressed in this Working Paper are those of the authors and do not necessarily reflect those of the Banco Central do Brasil. Ricardo Brito additionally thanks the financial support of CNPq grant n. 310360/2016-1.

1. Introduction

The documentation of the U.S. equity premium in the past century is comprehensive, and numbers like *8%* annual equity return, above *6%* annual equity premium, below *20%* volatility, and *0.40* Sharpe ratio are at the top of the head of every financial economist. Studies that contemplate other countries' experiences for 50 years or more also exist for other industrial economies (see Campbell, 2003, and Dimson et al., 2008), but are scant for emerging economies.

Estimates of long-term returns and appraisals of their magnitudes through the lens of theory in different environments are key inputs driving asset pricing research and portfolio management. Whether the above magnitudes could result from micro-founded theories of rational investment under uncertainty has been the motif of a vast literature since Mehra and Prescott (1985). In summary, researchers regard the U.S. historical market stock returns as puzzlingly high (see Mehra and Prescott, 2003).[1]

However, how do the equity returns of a typical emerging economy compare with the U.S. from a half-century perspective? What are the challenges for explaining observed data for an emerging economy according to standard asset pricing theory?[2]

In this paper, we document the 1968-2019 equity premium in Brazil, where macroeconomic risks have been substantial, providing an opportunity to learn about asset pricing

[1] For other industrial economies, Dimson et al. (2008) find high Sharpe ratios, though less impressive than in the U.S.; and Campbell (2003) concludes that implicit risk aversions from the consumption-based model are implausibly high in general.

[2] Goetzmann and Ibbotson (2008) point out that "… Our understanding of the historical experience of investors is relatively limited once we step beyond a few well-studied markets."

in emerging markets.[3] We assess the Brazilian experience against the U.S., which serves as the benchmark economy. The comparative analysis encompasses several dimensions, concentrating on the following issues: (a) average returns and equity premium estimated under alternative methods; (b) performance measures that consider higher-order moments; (c) long-term expected returns over different investment horizons; and (d) long-run asset allocation between risky and riskless assets. This multidimensional viewpoint gives a broader picture of the functioning of the stock market in a typical emerging economy, in contrast to industrial countries' experiences.[4]

To the best of our knowledge, we are the first to study the Brazilian equity premium and long-term returns for such an extended period, which begins with the creation of the Ibovespa – the São Paulo Stock Exchange index.[5] Previous literature, limited by data availability, does not provide this long-term perspective and disagrees on the attractiveness of Brazilian equity returns

3 The high Brazilian stock market volatility pops out in international comparisons. For example, it is the second highest in Fama and French (1998), below Argentina, and it is the third highest in Rouwenhorst (1999), below Argentina and Venezuela.

4 According to the World Factbook from the U.S. Central Intelligence Agency (https://www.cia.gov/library/publications/the-world-factbook/geos/br.html), in 2017, Brazil produced a GDP (purchasing power parity) of US$3.24 trillion (eighth largest economy in the world) with a population of 207 million. The Brazilian market value of publicly traded shares was $642.5 billion on 31 December 2017 (nineteenth largest in the world).

5 The São Paulo Stock Exchange (Bovespa) is nowadays part of B3 (in full, B3 - Brasil Bolsa Balcão S.A. or B3 - Brazil, Stock Exchange and Over-the-Counter Market). In 2008, the Bovespa and the Brazilian Mercantile and Futures Exchange (BM&F) merged, creating BM&FBOVESPA. In 2017, BM&FBOVESPA merged with CETIP, creating B3. There were 338 companies listed at Bovespa as of March 2017.

(see alternatives in Fama and French, 1998, Rouwenhorst, 1999, Bonomo and Garcia, 2001, Cysne, 2006, or Varga and Brito, 2016).[6]

Concerning ex-ante returns, the arithmetic mean real return of the Brazilian stock market is 21.3% per year from 1968 to 2019. The equity premium over savings account interest rates is 20.1% per year, with a huge standard deviation of 67%, implying a Sharpe ratio of 0.30. For the same period, the arithmetic mean return for the U.S. stock market is 8.2% per year. The U.S. equity premium over 1-year Treasury Bills is 6.4% per year, with a standard deviation of 17%, implying a Sharpe ratio of 0.38. Alternatively, continuous compounding reveals a Brazilian geometric mean equity return of 6.8% per year with 49.2% annual volatility and a mean risk-free rate of 0.86% per year. For the U.S., the geometric mean equity return is 6.3% per year with 17.8% annual volatility, and a mean risk-free rate of 1.59% per year.[7]

We additionally analyze percentiles, higher moments and the Aumann and Serrano (2008) riskiness index (AS index henceforth) to improve our appreciation of how the returns distributions differ in moments that matter to investors. The AS index clearly indicates the much higher risk of

[6] Fama and French (1998) compute an average annual dollar return of the Brazilian market, in excess of the U.S. T-Bill between 1987 and 1995, equal to 34.99% (with 79.15% standard deviation). Rouwenhorst (1999) finds a Brazilian market return of 19.35% (with 26.67% standard deviation) in local currency for the period 1982:Q1-1997:Q4, or of 4.27% (with 20.17% standard deviation) in US Dollars. Bonomo and Garcia (2001) document an average equity premium of 28.82% (with 70.49% standard deviation) for the period 1976:1-1992:12. Cysne (2006) presents an average Brazilian market return of 31.33% and equity premium of 15.92% over the Brazilian interbank rate between 1992 and 2004. Varga and Brito (2016) show an average monthly market return of 1.08% (with 7.84% standard deviation) between 1999:7 and 2015:6.

[7] Section 2.4 explains how the arithmetic and geometric means can provide such deceptively different pictures. Anticipating, given lognormality of the discrete gross rates of return, R : $lnE(R) = E(lnR) + 0.5\sigma^2(lnR)$.

the Brazilian markets relative to their U.S. equivalents. However, we surprisingly conclude that the skewness and kurtosis effects on the AS indices for both countries are not sizeable and are similar. In sum, the main difference in the riskiness of these countries' stock markets is in the enormous variance of Brazilian returns, with significant implications for expected returns and asset allocation.

While the difference between the arithmetic and geometric population means is well understood, the bias generated in compounding with one or the other sample average depends on the interaction between market volatility and investment horizon, with non-trivial effects on expected long-term returns. Using the unbiased estimator suggested in Jacquier et al. (2003), we account for the impact of the mean parameter uncertainty under the distinct Brazilian and U.S. volatility environments.[8] Because of its much higher volatility, Brazilian long-term expected stock returns are considerably penalized.

Finally, we show that the dissimilar Brazilian and U.S. stock market returns can result from the demands of investors that handle risk similarly. Although there are striking differences in the macroeconomic environments and resulting volatilities that impact on stock holdings and cost of equity, Brazilian and U.S. investors can be depicted as being alike. The extension of Merton's (1969) "lifetime portfolio" model, which takes sampling uncertainty into account as in Jacquier et al. (2005), rationalizes both Brazilian and U.S. numbers with similar risk aversions. Although equity returns and the equity premium have been higher in Brazil than in the U.S., the much higher Brazilian equity volatility discourages heavier investments in stocks. For similar relative risk

[8] Given our historical perspective of the equity premium, we abstract from the literature on predictability (Campbell and Thompson, 2008, Welch and Goyal, 2008), learning (Barberis, 2000), or model-implied forward-looking premium (Jagannathan et al., 2000, Fama and French, 2002).

aversions in the neighborhood of 4 to 6, the model implies equity allocations close to *12%* and *32%* of financial wealth respectively in Brazil and the U.S., matching income tax data. [9]

In the following section, we present brief histories of stock returns in Brazil and the U.S. during the past fifty-two years from the perspective of three estimators: arithmetic mean, geometric mean, and an unbiased alternative. In section 3, we analyze both markets through the lens of Merton's (1969) lifetime portfolio model with expected returns uncertainty. We conclude in section 4.

2. Two 50-year Histories

2.1 Data

We study the Ibovespa, a total return index of the São Paulo Stock Exchange (Bovespa), from its creation in December of 1967 until December of 2019. The Brazilian market return series are nominal, and we deflate them by the General Price Index (*Índice Geral de Preços – Disponibilidade Interna*, IGP-DI). Concerned with the domestic investor view, we compute returns in the local currency. [10]

[9] If instead of Merton's (1969) terminal wealth perspective, we choose the consumption-based asset pricing approach of Mehra and Prescott (1985), we find that the equity premium is as puzzlingly high in Brazil as it is in the U.S. for reasonable degrees of risk aversion. That is, risk aversions in both countries have to be very high to accommodate such premia, which is another sign of the similarity between Brazilian and U.S. investors. This "short-term investment" perspective is available upon request in a longer working paper version.

[10] Because we are interested in real returns, we do not report nominal statistics. For those curious about nominal returns, we can provide the respective tables upon request.

We choose the return on the Savings Account, called *Caderneta de Poupança,* as the Brazilian "riskless" short-term real interest rate series. The *Caderneta de Poupança* is (imperfectly) inflation-indexed. It is the most popular financial investment vehicle in Brazil and regarded as the least risky investment by individuals. It is presented in the tables below under the label of *Short-term real interest* . Alternative interest rates used in the Brazilian literature are not available for this extended period and embed varying bank spreads. [11]

For the U.S., like Dimson et al. (2008), we use the capitalization-weighted CRSP Index of all NYSE stocks from 1968 to 1970. Thenceforth, from 1971 to 2019, we employ the Wilshire 5000 Index, which contains over 7,000 U.S. stocks, including those listed on Nasdaq. We deflate U.S. nominal series by the Producer Price Index for All Commodities. As for the short-term interest rate, we use the 1-year constant maturity U.S. Treasury Bill rate.

To keep an eye on countries' real activity, we also present annual real Gross Domestic Product (in constant LCU) growth rates. Both the Brazilian and U.S. series are from the World Bank.

[11] In a previous version of this paper, we combined two short-term interest rate series: (i) the return on the Savings Account; and (ii) a merge of the Brazilian Treasury Obligations (*Obrigações do Tesouro Nacional and Obrigações Reajustáveis do Tesouro Nacional* until 1974) with the Brazilian interbank rate (*SELIC* after 1974). Such composite series results in a higher average short-term interest rate. However, presentations and discussions made us forgo this option. These previous results are available upon request with the same conclusions of this current version. Facing a similar choice between U.S. government and U.S. municipals in the 19th century, Goetzmann and Ibbotson (2008) choose the minimum yield between yearly U.S. government and U.S. municipals as a measure of the (nearly) riskless rate.

2.2 Arithmetic averages

What are the expected real equity and short-term interest returns in Brazil? And how does the Brazilian equity premium compare with that in the U.S.?

Academics, concerned with ex-ante expected returns, advocate using the arithmetic mean. For example, Mehra and Prescott (2008) define V_H as the value H periods into the future:

$$V_H = V_0 \prod_{t=1}^{H} R_t = V_0 \prod_{t=1}^{H} (1 + a_t), \tag{1}$$

where V_0 is the amount invested today and $R_t = (1 + a_t)$ is period-t realized return.

If one takes expectations and assumes uncorrelated returns:

$$E[V_H] = V_0 \prod_{t=1}^{H} (1 + E[a_t]) = V_0(1 + \bar{a})^H = V_0 e^{\bar{\alpha} H}, \tag{2}$$

where $\bar{a}$ is the arithmetic mean rate of return and $\bar{\alpha}$ is its continuously compounded-return equivalent.

In Table I, we present arithmetic sample averages ($\hat{a}$) of real returns and other summary statistics of key financial and macro variables for the 1968-2019 period. The average real returns on stocks are high. The mean return of the Brazilian stock market is *21.3%* per year with volatility of *66.6%*, while the U.S. mean stock market return is *8.2%* per year with volatility of *18.1%*. We cannot reject the hypotheses that these stock markets' annual returns are not autocorrelated (tests results presented in Table II below). Figures 1.1 and 1.2 illustrate how different the histograms of equity returns are, identifying each year's return in the distributions.

Table I - Annual returns - 1968-2019

	Mean	Std. Dev.	Minimum	Maximum	N. of negatives	Longest run of negatives *
	(1)	(2)	(3)	(4)	(5)	(6)
			Brazil			
Equity	0.2127	0.6656	-0.7411 ('90)	3.1638 ('91)	25	6 ('10)
Short-term interest	0.0115	0.0749	-0.2368 ('80)	0.2230 ('95)	22	4 ('99)
Equity premium	0.2012	0.6704	-0.7605 ('87)	3.2059 ('91)	24	3 ('72;'78;'00;'13)
GDP growth	0.0388	0.0426	-0.0439 ('81)	0.1398 ('73)	8	2 ('15)
Inflation	2.2656	5.2850	-0.0143 ('09)	27.0817 ('93)	2	
			U.S.A.			
Equity	0.0817	0.1811	-0.4070 ('74)	0.3750 ('91)	18	3 ('00)
Short-term interest	0.0174	0.0534	-0.1126 ('74)	0.1228 ('01)	21	5 ('09)
Equity premium	0.0642	0.1696	-0.4232 ('08)	0.3281 ('13)	15	3 ('00)
GDP growth	0.0284	0.0194	-0.0278 ('09)	0.0726 ('84)	7	2 ('74; '08)
Inflation	0.0360	0.0512	-0.0685 ('15)	0.2089 ('74)	9	

Notes: Annually compounded rates per year in the respective local currency. Returns are real returns, except for inflation. Equity premium is the Equity return minus the return on Short-term interest. Std.Dev. is the standard deviation of the annually compounded returns. Computed using *52* yearly observations. Number in parentheses indicates the year of occurence.
* In column (9), the number in parentheses indicates the first year of the sequence of years.

Figure 1.1 - Histogram of annual Brazilian real equity returns, 1968-2019

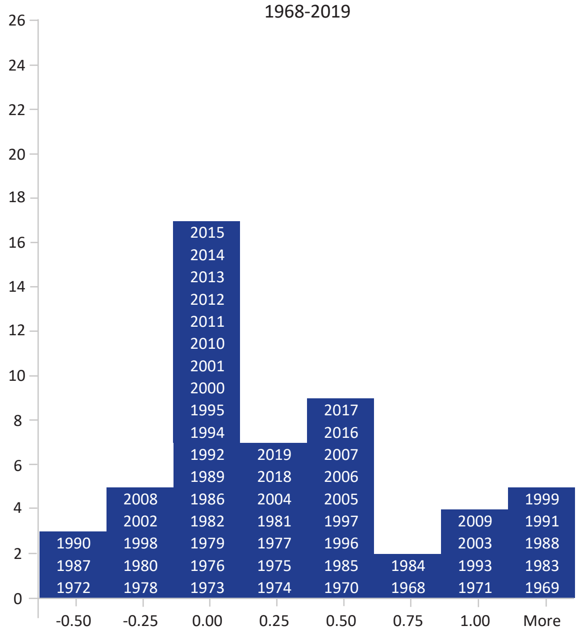
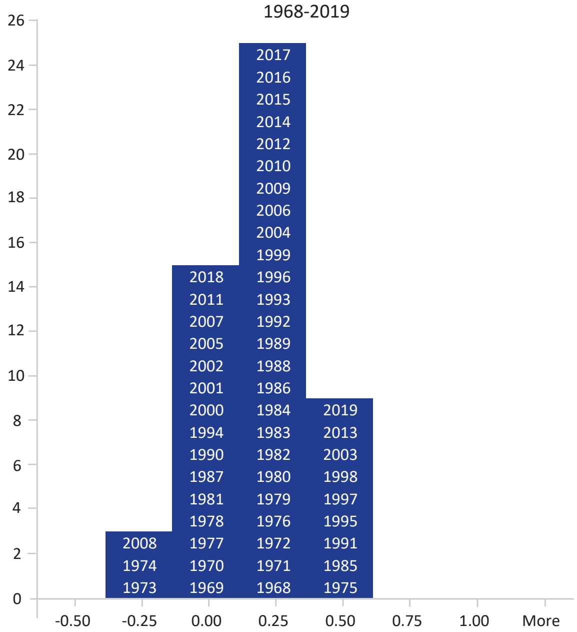


Figure 1.2 - Histogram of annual US real equity returns, 1968-2019



Relative to stocks, the short-term real interest rates are low and much less volatile. From those numbers, a Brazilian equity premium of *20.1%* per year emerges, with a standard deviation of *67.0%* and a Sharpe ratio of *0.30* . The U.S. equity premium is *6.4%* per year with a standard deviation of *17.0%* and a Sharpe ratio of *0.38* .

Readers aware of the Brazilian fixed-income market reputation for paying high real interest rates may question our picture with an average annual “riskless” real return of *1.15%* , which is below the notoriously low U.S. annual average of *1.74%* . Another concern is about the certainty (or riskiness) of real short-term interest in the face of the Brazilian high inflation experience. We point out that, although unpredictable shocks to inflation are more important in Brazil than in the U.S., the *-0.17* correlation between real interest rate and inflation in Brazil is weaker than the respective correlation of *-0.78* in the U.S. (correlation numbers not presented in the tables). We argue that this lower real return-weaker inflation correlation configuration of the Brazilian interest rates is reasonable, given the inflation-indexed nature of the Brazilian Savings Account that, by (imperfectly) insuring against the significant inflation risk, thus pays a lower return. Particularly empirically convenient, this indexation considerably offsets the Brazilian high-inflation distortions on a supposedly “riskless” real interest rate.

Considering real economic activity, the Brazil x U.S. differences in terms of GDP growth and inflation are also evident in the rows of Table I. Both GDP growth rate averages are lower than their respective average stock returns and higher than their short-term interest rates. The Brazilian GDP growth rates have been higher on average and much more volatile than those for the U.S. economy.

In this fifty-two-year period, Brazil went through years of high economic volatility with the exhaustion of a cycle of high growth accrued from its industrialization, mainly funded by public savings. Deadlocks in the simultaneous re-democratization process and lack of consensus over the macroeconomic agenda degenerated into a severe fiscal crisis, and more than a decade to tame a very high and persistent inflation.[12] The average inflation in this half-century was 227% per year, mostly accrued in the 1980s and early 1990s – the 1980-1994 average is 746% per year, with annual rates as high as 1783% in 1989, 1477% in 1990 and 2708% in 1993.[13] There were seven major stabilization plans between 1986 and 1994, which tried measures such as price

12 Between 1968 and 2019, Brazil had twelve presidents: four Army generals (until 15-Mar.1985) and one civilian all selected indirectly (until 15-Mar.1990), and five elected in general democratic elections. Among the latter five, two were impeached (Fernando Collor on 29-Dec.1992 and Dilma Rousseff on 17-Apr.2016) and succeeded by their vice-presidents.

13 Brazil underwent six monetary reforms in the 1968-2019 period. The local currencies were Cruzeiro Novo (13-Feb.1967), Cruzeiro (15-May.1970), Cruzado (28-Feb.1986), Cruzado Novo (16-Jan.1989), Cruzeiro (16-Mar.1990), Cruzeiro Real (1-Ago.1993) and Real (since 1-Jul.1994).

controls, external debt moratorium, financial assets freezes, prohibition of indexed contracts, government spending controls, and exchange rate anchor. [14]

This long inflation struggle had marked real effects. The Brazilian stock market had its worst years in 1990 and 1987, down by *74.11%* and *73.85%* respectively, and coinciding with the failures of two main inflation stabilization attempts: "Plano Cruzado" and "Plano Collor". Primarily a recovery from the 1990 stocks' fire sale, and partially due to the worldwide increase in business optimism, 1991 was the Brazilian stock market's best year, with an impressive return of *316.38%*.

In comparison, in the U.S. the extreme years were the consequence of real shocks. The worst year was 1974, down by *40.70%* attributed to a combination of the 1973 oil crisis and the collapse of the Bretton Woods system over the previous years. [15] Corroborating the worldwide increase in business optimism, the U.S. also had its best year in 1991, with the stock market going up by *37.50%*.

Table I additionally details how the higher Brazilian volatility outshines realized equity returns. The Brazilian minimum and maximum, respectively in columns (3) and (4), as well as the number of year with negative returns in column (5), provide concreteness to the much higher risk in Brazilian equities. Out of the fifty-two years studied, the Brazilian and U.S. stock markets had respectively *25* and *18* negative real returns. The longest sequence of negative stock returns took

[14] Before Plano Real in 28-Feb.1994, which finally reduced inflation to one digit on average (the average inflation between 1995-2017 was 8.32% per year), there were Plano Cruzado (28-Feb.1986), Plano Cruzado 2 (22-Nov.1986), Plano Bresser (12-Jun.1987), Plano Verão (12-Jan.1989), Plano Collor 1 (16-Mar.1990) and Plano Collor 2 (31-Jan. 1991).

[15] The U.S. stock market went down by *29.51%* in 1973.

six years in Brazil, between 2010 and 2015 and three years in the U.S., between 2000 and 2002 (see column (6)).

Despite the much higher Brazilian volatility, the correlations of 0.23 between countries' GDPs and of 0.33 between countries' stock returns are evidence of an important common world business activity factor (correlation numbers not presented in the tables).

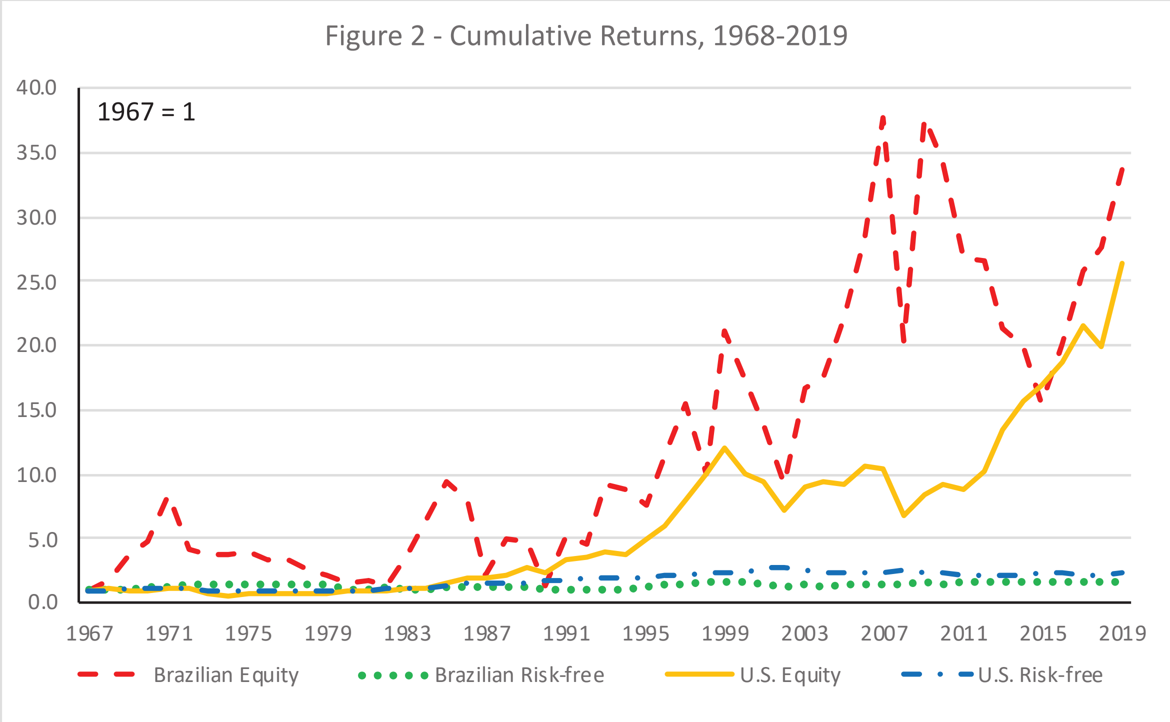
2.3 Geometric average

Practitioners prefer the simple intuition of compounding:

$$\left(\frac{V_H}{V_0}\right) = e^{\hat{r}H} \qquad \Longrightarrow \qquad \hat{r} = \frac{1}{H} ln\left(\frac{V_H}{V_0}\right), \tag{3}$$

where $\hat{r}$ is the geometric average, as the clearest way to represent wealth growth observed in the past. Additionally, geometric averages produce lower, or more conservative, long-term forecasts than arithmetic averages.

Figure 2 displays cumulative returns during the past fifty-two years, and Table II presents descriptive statistics of geometric annualized returns – i.e., continuously compounded annualized rates – for our variables of interest computed from quarterly data.



The first two rows of Table II present annualized geometric averages and standard deviations from quarterly data. The Brazilian geometric average equity return is *6.77%* per year with a very high *49.21%* volatility; and the short-term interest rate average is *0.86%* per year with *7.19%* volatility. With much lower equity volatility of *17.76%*, the U.S. geometric average equity return is *6.30%* per year; and the short-term interest rate has an average of *1.59%* per year and volatility of *4.09%*.

Table II - Geometric mean annual rates - 1968-2019

	Brazil			*U.S.A.*		
	Equity real return	Short-term real interest	Equity premium	Equity real return	Short-term real interest	Equity premium
	(1)	(2)	(3)	(4)	(5)	(6)
Mean	0.0677	0.0086	0.0591	0.0630	0.0159	0.0471
Std.Dev.	0.4921	0.0719	0.5009	0.1776	0.0409	0.1730
Skewness	-0.5062	-0.4228	-0.4162	-0.3735	0.4679	-0.4327
Kurtosis	4.7053	3.8488	4.4245	3.3672	5.5159	3.3373
25th-percentile	-0.3964	-0.0552	-0.4335	-0.0803	-0.0241	-0.1257
Median	0.0898	0.0230	0.0996	0.1028	0.0233	0.1027
75th-percentile	0.5624	0.0873	0.5462	0.2819	0.0576	0.2473
AS riskiness index	1.888	0.312	2.204	0.275	0.051	0.346
.5*(Std.Dev.^2)/Mean	1.789	0.301	2.122	0.251	0.052	0.320
Ratio	1.055	1.037	1.038	1.096	0.993	1.080
Minimum 5-year	-0.4027	-0.0759	-0.3759	-0.1333	-0.0375	-0.5062
Negatives in 5-yrs	19	16	22	15	19	13
Minimum 10-year	-0.1631	-0.0293	-0.1495	-0.0444	-0.0249	-0.0418
Negatives in 10-yrs	11	16	12	9	16	8
Minimum 20-year	-1.3308	-0.2955	-1.0901	0.6476	0.0031	0.2425
Negatives in 20-yrs	2	7	8	0	0	0
Minimum 25-year	0.0120	-0.0058	0.0107	0.0503	0.0069	0.0253
Negatives in 25-yrs	0	5	0	0	0	0
H_0: Autocorrelated	0.26	0.42	0.10	0.52	0.00	0.55

Notes: Continuously compounded annualized rates computed from quarterly data from 1968:Q1-2019:Q4 (*208* observations) in the respective local currency. Skewness is the third moment about the Mean divided by *Std.Dev.^3* . Kurtosis is the fourth moment about the Mean divided by *Std.Dev.^4* . *AS* is the Aumann-Serrano (2008) index of riskiness from the normal inverse Gaussian distribution and *.5*(Std.Dev.^2)/Mean* is the value to which the AS index degenates when the data are normally distributed. Ratio is the former divided by the latter. The Cumby-Huizinga test for autocorrelation reports the *p* -value.

Skewness, kurtosis and percentiles in Table II provide complementary information for those forecasting future returns.[16] Both Brazilian and U.S. quarterly stock returns are negatively skewed with positive excess-kurtosis (i.e., kurtosis minus 3). In Brazilian stocks, there is a 25% probability of getting a quarterly return lower than -9.1% (or, annualized -39.6%) and a 25% probability of a quarterly return higher than 14.1% (or, annualized 56.2%). With a less spread-out distribution for the U.S. stock returns, these numbers are -2.0% (or, annualized -8.0%) for the 25th-percentile and 7.0% (or, annualized 28.2%) for the 75th-percentile. Notice that the Brazilian median is below the U.S. median. That is, with 50% probability, Brazilian stocks return less than 2.2% per quarter (i.e., 9.0% annualized), while U.S. stocks return less than 2.6% per quarter (i.e., 10.3% annualized).

Although the descriptive statistics listed in the above paragraphs make clear that the Brazilian stock market is more volatile than the U.S. market, they do not provide an objective measure of riskiness. Another appraisal missing is how these returns depart from the Normal distribution. Long-horizon continuously compounded returns converge to normal distributions, but that is not yet the case for quarterly returns (see Cont, 2001, and Fama and French, 2018b).

Aumann and Serrano (2008) propose an index of "riskiness" that addresses these two issues. The AS index enables an investor to assess which of two investments is *riskier* without referring to a specific utility function, thereby making comparisons easy. Although it is not our objective to put the Brazilian and U.S. stock markets as alternatives to the same investor, it is informative to assess the relative riskiness of the two markets through a riskiness index.

[16] See Hughson et al. (2006) for an argument of why investors should be more interested in medians and percentiles than in the mathematical expectation. Harvey and Siddique (2000) and Dittmar (2002), among others, demonstrate the importance of skewness and kurtosis in investor preferences.

Additionally, the AS index accounts for higher moments of the returns distributions and provide a measure of how they are far from Normal.

From the normal inverse Gaussian distribution, Homm and Pigorsch (2012) provide the following parametric formula for the AS index:

$$AS^{NIG}(\mu, \sigma^2, \chi, \kappa) = (3\kappa\mu - 4\mu\chi^2 - 6\chi\sigma + 9\sigma^2/\mu)/18, \tag{4}$$

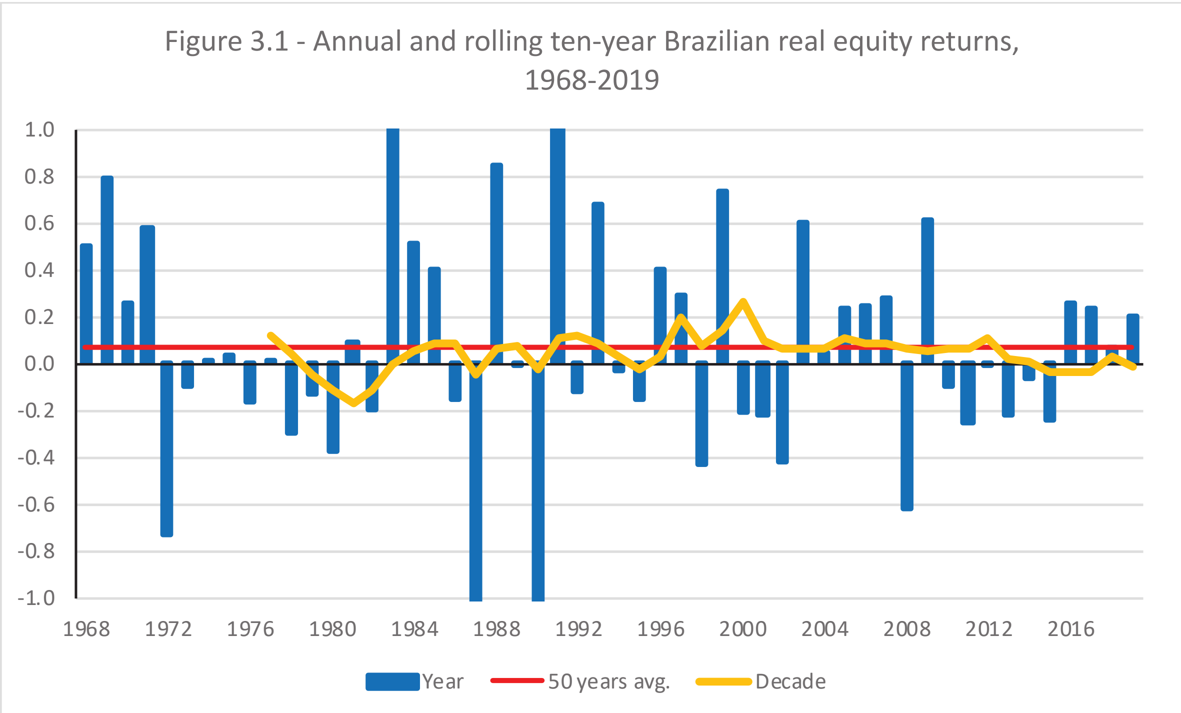
where μ is the mean, σ^2 is the variance, χ is the skewness and κ is the excess-kurtosis. In case skewness and excess-kurtosis are zero, the return distribution converges to the normal distribution, and the AS index becomes:

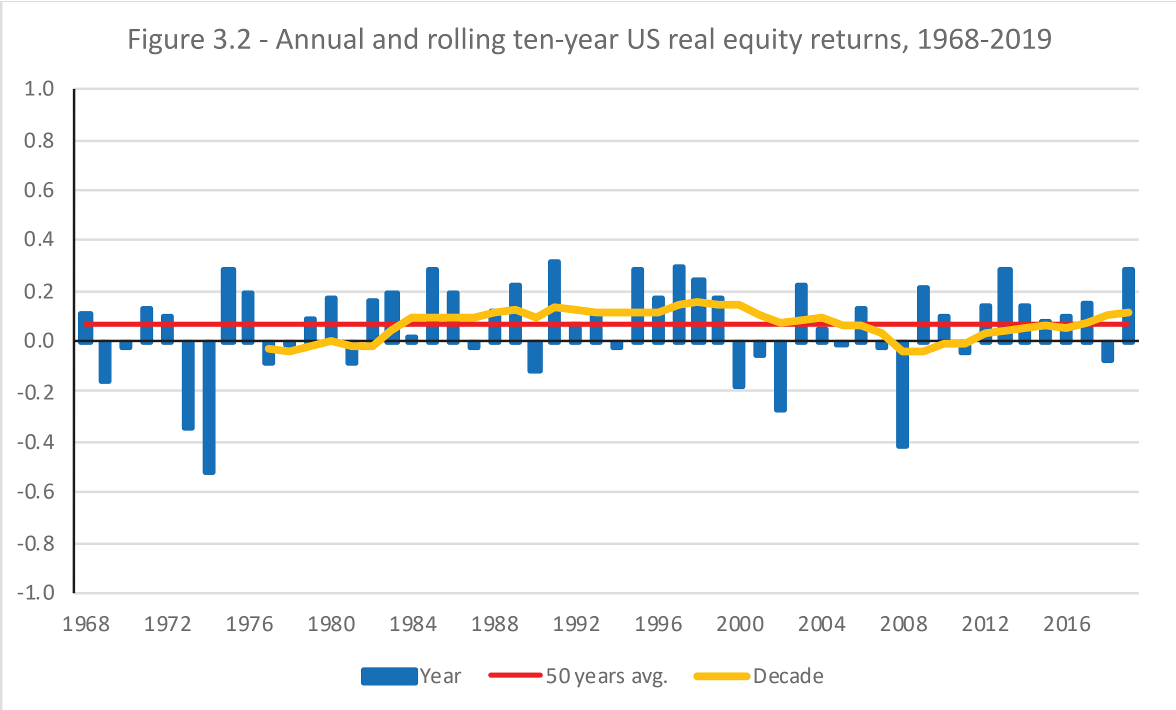
$$AS^{Normal}(\mu, \sigma^2, \chi, \kappa) = (1/2)(\sigma^2/\mu). \tag{5}$$

The AS indices in Table II confirm that the Brazilian markets are much riskier than the U.S. markets. However, the Brazilian returns distributions are not further from the Normal distribution than the U.S. returns distributions. Actually, the Brazilian *Ratio*s of equations (4) to (5) are closer to *1* than the U.S. *Ratio*s, indicating that the higher riskiness of the Brazilian markets derive mostly from its high variances.

Table II also presents minimum cumulated returns in 5-, 10- and 25-year windows. In addition, it shows the numbers of rolling windows, within the 52 years studied, in which the investment resulted in negative cumulative returns after investing for that respective horizon. For example, in column (1) of row *Negatives in 10-yrs*, the "*11*" means there were eleven specific years in which Brazilian stocks produced cumulative losses after ten-year investments. One can identify those years along the yellow line in Figure 3.1. Respectively for Brazil and the U.S., Figures 3.1 and 3.2 illustrate the realized annual geometric equity returns for a rolling decade, the full 52-year period, and on a year-by-year basis.

While stocks have fewer years of negative real returns than the riskless short-term interest in the U.S., it is the opposite in Brazil, due to the latter high stock market volatility. Note, however, that as the investment horizon increases, the equity risk of loss decreases relative to that of the short-term real interest rate in both markets.





Finally, Table II presents *p*-values of the Cumby-Huizinga test that do not reject the hypothesis of no autocorrelation. Uncorrelated returns is a key assumption to infer expected values from historical averages, as has been suggested in this paper.

2.4 An unbiased long-term mean return estimator

Assuming returns are lognormally distributed $lnR = r \sim N(\bar{r}, \sigma^2)$ – an assumption that gets better as the horizon increases, according to Cont (2001) and Fama and French (2018b) [17] – the Brazilian much higher volatility than that for the U.S. explains why the large difference

[17] Normality tests usually reject that quarterly continuously compounded returns are normally distributed. However, distributions of continuously compounded returns converge toward normal distributions with horizon extension from one to 30 years, as demonstrated in Fama and French (2018b).

between their arithmetic means in Table I shrinks when we look at geometric returns in Table II. It should be that:

$$(1+\bar{a}) = e^{\bar{\alpha}} = e^{\bar{r}+\frac{1}{2}\sigma^2}. \tag{6}$$

where the $\frac{1}{2}\sigma^2$ term converts the expected return from a geometric mean to an arithmetic mean. That is a Jensen's Inequality adjustment, since we are describing expectations of log returns: $lnE(R) = E(lnR) + 0.5\sigma^2(lnR)$. Note that the Brazilian stock market volatility is so much higher than the U.S. that $\left(\bar{r}+\frac{1}{2}\sigma^2\right)/\bar{r} = 2.79$ in Brazil and $\left(\bar{r}+\frac{1}{2}\sigma^2\right)/\bar{r} = 1.25$ in the U.S..

The "Arithmetic" column of Table III computes average returns from Equation (6), and provides a sense of the lognormal assumption. These values are similar to their equivalents in column (1) of Table I, and thus henceforward we use the geometric average and standard deviation to build expected rates of return.

However, Jacquier et al. (2003) recall that mean estimates are subject to sampling variation. For lognormal returns, the geometric average estimate is:

$$\hat{r} = \bar{r} + \varepsilon\frac{\sigma}{\sqrt{T}}, \qquad \varepsilon \sim N(0,1), \tag{7}$$

where T is the time span of the sample used in the estimation. Thus, the estimated return of an investment with horizon H is:

$$e^{\left(\hat{r}+\frac{1}{2}\sigma^2\right)H} = e^{\left(\bar{r}+\varepsilon\frac{\sigma}{\sqrt{T}}+\frac{1}{2}\sigma^2\right)H} = e^{\left(\bar{r}+\frac{1}{2}\sigma^2\right)H}e^{\left(\varepsilon\frac{\sigma}{\sqrt{T}}\right)H}, \tag{8}$$

and the expected estimate is:

$$E\left[e^{\left(\hat{r}+\frac{1}{2}\sigma^2\right)H}\right] = e^{\left(\bar{r}+\frac{1}{2}\sigma^2\right)H} E\left[e^{\left(\varepsilon\frac{\sigma}{\sqrt{T}}\right)H}\right] = e^{\bar{\alpha}H} e^{\left(\frac{1}{2}\sigma^2\frac{H^2}{T}\right)}, \tag{9}$$

showing that the arithmetic mean estimates are biased upward by the last term, $e^{\left(\frac{1}{2}\sigma^2\frac{H^2}{T}\right)}$.

Alternatively, one can write:

$$E\left[e^{\hat{r}H}\right] = E\left[e^{\left(\bar{r}+\varepsilon\frac{\sigma}{\sqrt{T}}\right)H}\right] = e^{\bar{r}H} e^{\left(\frac{1}{2}\sigma^2\frac{H^2}{T}\right)} = (1+\bar{\alpha})^H e^{\frac{1}{2}\sigma^2\left(\frac{H}{T}-1\right)H}, \tag{10}$$

which indicates that the geometric mean estimates are biased downward if $H<T$, and the bias increases with the volatility.

To remove such bias in the expected rates of return, Jacquier et al. (2003) suggest compounding at the unbiased mean rate-of-return estimator:

$$\hat{\alpha}^{unb} = \hat{r} + \frac{1}{2}\sigma^2\left(1-\frac{H}{T}\right), \tag{11}$$

which has expectation:

$$E\left[e^{\left(\bar{r}+\varepsilon\frac{\sigma}{\sqrt{T}}+\frac{1}{2}\sigma^2\left(1-\frac{H}{T}\right)\right)H}\right] = e^{\left(\bar{r}+\frac{1}{2}\sigma^2\left(1-\frac{H}{T}\right)\right)H} e^{\left(\frac{1}{2}\sigma^2\frac{H^2}{T}\right)} = e^{\left(\bar{r}+\frac{1}{2}\sigma^2\right)H}. \tag{12}$$

Jacquier et al. (2005) additionally propose an alternative "small-sample efficient" estimator, which minimizes the RMSE and presents significant efficiency gains. However, they abstract from its bias side-effect on the expected future portfolio value. Although the RMSE gain of the small-sample efficient estimator over the arithmetic mean is significant, the small-sample efficient estimator bias is sizeable, the reason why we advocate the unbiased estimator. [18]

In Table III, we present the unbiased mean returns for horizons from one to twenty-five years. As the horizon H increases, expected annual returns decrease faster in Brazilian equities, where the volatility is much higher. Note that the Brazilian-to-U.S. equity return ratio decreases from 2.51 at the 1-year horizon to 1.89 at the 25-year horizon. On the other hand, the least volatile U. S. GDP growth is almost unaffected. Intuitively, because of uncertainty about the mean return parameter, an investor considering different horizons formulates different point forecasts.

[18] Although the RMSE gain of the small-sample efficient estimator over the arithmetic mean is of *36%* for Jacquier et al. (2005), choosing *H/T=25/60* , mean $\mu = 0.10$ and volatility $\sigma = 0.20$, the small-sample efficient estimator bias amounts to *-34%* of the unbiased expected future portfolio in *H=25* periods. In their notation, the bias formula is: $[E(C)/E(V_H)] = exp\{0.5\sigma^2 H[k + (H/T) - 1]\}$ where C is the estimator. For $k = 1 - 3(H/T)$ of the small-sample efficient estimator, we get to a bias of $[E(C)/E(V_H)] = exp\{-\sigma^2(H^2/T)\}$. With the Brazilian parameters and *H/T = 25/50* , the small-sample efficient estimator bias amounts to *-95%* of the unbiased expected future portfolio in *H=25* periods.

Table III - Unbiased mean annual real returns for different horizons - 1968-2019

	"Arithmetic"	Horizon (H in years)				
		1	5	10	20	25
		Brazil				
Equity return	0.2078	0.2050	0.1938	0.1800	0.1528	0.1395
Short-term interest	0.0112	0.0112	0.0110	0.0107	0.0102	0.0100
GDP growth	0.0388	0.0388	0.0387	0.0386	0.0384	0.0384
		U.S.A.				
Equity return	0.0820	0.0816	0.0803	0.0787	0.0754	0.0738
Short-term interest	0.0169	0.0169	0.0168	0.0167	0.0166	0.0165
GDP growth	0.0284	0.0284	0.0284	0.0283	0.0283	0.0283

Notes: Annually compounded real rates expressed in % per year. The "Arithmetic" is *{exp[Geometric Mean + .5*(Std.Dev.^2)]-1}* . For horizon *H* , the unbiased mean annual real return is *{exp[Geometric Mean + .5*(Std.Dev.^2)*(1-(H/52)]-1}* . Computed using *208* quarterly observations.

Analogously, one could ask about the size of expected cumulated wealth. We provide this information in Table IV. The investment of *$1* in the Brazilian stock market is expected to return *$26.16* after 25 years, while *$1* in the U.S. stock market for 25 years is expected to return *$5.93*.

Table IV - Unbiased mean terminal wealth (in multiples of initial) for different horizons - 1968-2019

	"Arithmetic"	Horizon (H in years)				
		1	5	10	20	25
				Brazil		
Equity return	1.21	1.20	2.42	5.23	17.19	26.16
Short-term interest	1.01	1.01	1.06	1.11	1.23	1.28
GDP growth	1.04	1.04	1.21	1.46	2.13	2.56
				U.S.A.		
Equity return	1.08	1.08	1.47	2.13	4.28	5.93
Short-term interest	1.02	1.02	1.09	1.18	1.39	1.50
GDP growth	1.03	1.03	1.15	1.32	1.75	2.01

Note: Terminal wealth after H years investment (V_H) of *$1* . "Arithmetic" for *H=1* is *exp[Geometric Mean + .5*(Std.Dev.^2)]* . For horizon H , the unbiased mean terminal wealth V_H = *exp{[Geometric Mean + .5*(Std.Dev.^2)*(1-(H/52)]*H}* . Computed using *208* quarterly observations.

We warn that, besides the positive sample mean bias, corrected in Tables III and IV above, investors should be aware of the asymmetric effect of volatility on the percentiles of the lognormal distribution. Fama and French (2018a) present convincing simulations that high volatility implies nontrivial probabilities of negative realized premiums even for 10- and 20-year periods. And such negative realizations actually happened in the past fifty-two year histories of Brazil and the U.S., as indicated in Table II. Because of the lognormal's positive skewness, Hughson et al. (2006) even argue that the median return is a better statistic than the mean (which is too optimistic) for those interested in forecasting future cumulative returns. [19]

Although these authors' perspectives are enlightening of relevant aspects of risk, their approaches do not prescribe a normative optimal allocation, which we hope for when comparing with observed allocations. If so, we choose to judge variance and lognormality through a risk-averse utility function in the next section.

3. Risk Aversion and Optimal Allocation

The very different equity return histories raise the question: are national investors similar in nature? Precisely, is it possible to reconcile such different equity return processes with similar risk preferences?

Merton’s (1969) optimal lifetime-portfolio selection under uncertainty prescribes different allocations in equities according to the expected premium-volatility trade-off, for a given aversion to risk. Instead, we input the historical premium-volatility trade-offs and observed national allocations to stocks into Merton’s (1969) optimal formula, to deduce the implied risk aversions in the respective countries.

[19] Kan and Zhou (2009) get to the point of combining the Hughson et al. (2006) warning with the Jacquier et al. (2005) bias correction to the lognormal distribution, deriving an unbiased median estimator equal to $e^{\left(\hat{r}-\frac{1}{2}\sigma^2\frac{H}{T}\right)H}$, where the penalty for a high variance is sizeable. In section 3, we choose to penalize the variance through a concave (risk-averse) utility function.

Following Jacquier et al. (2005), we adapt Merton's (1969) problem to the context in which the expected equity return $\bar{r}_e$ is estimated.[20] The investor maximizes the expectations of her utility of final wealth in H periods from today, given the dataset $\mathcal{D}$:

$$E[U(V_H)|\mathcal{D}] = E\left[\frac{V_H^{1-\gamma}}{1-\gamma}|\mathcal{D}\right] = E\left[\frac{1}{1-\gamma}exp\{(1-\gamma)ln(V_H)\}|\mathcal{D}\right], \qquad (13)$$

subject to:

$$V_{t+1} = \left[1 + \bar{r}_f + w\left(r_{e,t+1} - \bar{r}_f\right)\right]V_t \,. \qquad (14)$$

The portfolio value H periods into the future is log-normal with parameters:

$$ln(V_H) \sim N(\alpha_H, \sigma_H^2) \equiv N\left(\left\{\begin{matrix}\left[\bar{r}_f + w\left(\bar{r}_e - \bar{r}_f\right)\right] \\ -\frac{1}{2}w^2\sigma_e^2\end{matrix}\right\}H, \qquad w^2\sigma_e^2 H\right). \qquad (15)$$

If we knew $\bar{r}_e$ for sure, the optimal allocation would be constant and independent of the horizon H : $w^* = \frac{\bar{r}_e + \frac{1}{2}\sigma_e^2 - \bar{r}_f}{\sigma_e^2\gamma}$. However, because we do not know $\bar{r}_e$, which we have to estimate with the dataset $\mathcal{D}$, the optimal allocation is:

$$w_H^* = \frac{\hat{r}_e + \frac{1}{2}\sigma_e^2 - \bar{r}_f}{\sigma_e^2\left[\gamma + \frac{H}{T}(\gamma - 1)\right]} \,. \qquad (16)$$

20 In the Appendix, we present an extended version where the expected short-term interest rate $\bar{r}_f$ is also estimated. For both Brazil and U.S., quantitative differences are small.

In the above equation, the $\frac{H}{T}(\gamma - 1)$ term comes from the sample variation of the estimated average $\hat{r}_e$, which amplifies the variance of $ln(V_H)$ by $\left(1 + \frac{H}{T}\right)$. Note that for $\gamma > 1$, w_H^* decreases with the investment horizon. The reasoning is, as estimation risk increases with the horizon H , equity allocations decrease proportionally with risk aversion. It is not just that investors formulate different point forecasts for different horizons, as illustrated in Table III. But investors with different risk aversions react differently to the horizon imprecision.[21]

Table V indicates the proportion allocated in equities for different risk aversions and horizons. For the same horizon and risk aversion (i.e, a bin in column H and row γ), the percentage of the wealth allocated in equities is lower in Brazil than in the U.S., this being rationalized by the much higher Brazilian equity volatility.

From the Financial Accounts of the United States, we find that the average participation of stocks in the total financial wealth of U.S. households is *0.323* between 1997 and 2016.[22] From

[21] Alternatively, one can rewrite (16) as:

$$w_H^* = \frac{\hat{r}_{eH}^* + \frac{1}{2}\sigma_e^2 - \bar{r}_f}{\sigma_e^2 \gamma},$$

where $\left(\hat{r}_{eH}^* + \frac{1}{2}\sigma_e^2 - \bar{r}_f\right) = \left(\hat{r}_e + \frac{1}{2}\sigma_e^2 - \bar{r}_f\right) - \left[\frac{\hat{r}_e + \frac{1}{2}\sigma_e^2 - \bar{r}_f}{\frac{\gamma}{\gamma - 1} + \frac{H}{T}}\right]\frac{H}{T}$ is the expected excess-return corrected for estimation risk, and frame the problem as if investors with different risk aversion formulate different point forecasts.

[22] The Financial Accounts of the United States are published by the Board of Governors of the Federal Reserve System, and are made available at FRED Economic Data. We compute the average participation of stocks in the total financial assets of Households and Nonprofits as the ratio of the sum of corporate equities and mutual fund shares to the difference between total financial assets and the liability in credit market instruments, i.e., (HNOCEAQ027S+HNOMFAA027N)/(HNOTFAA027N-TCMILBSHNO).

Afonso (2014), which uses proprietary data from the Brazilian Revenue Service, we calculate the average participation of stocks in the total financial wealth of Brazilian households to be close to *0.124* between 2005 and 2012.[23]

Table V - Optimal weights allocated to equity for different horizons

	Mean	Std. Dev.	γ	With known parameters	Horizon (H in years)				
					1	5	10	20	25
					Brazil				
Market index	0.0677	0.4921	**2**	0.372	0.368	0.355	0.339	0.312	0.300
Short-term interest	0.0086		**4**	0.186	0.183	0.173	0.163	0.144	0.137
			5	0.149	0.147	0.138	0.129	0.114	0.107
			6	0.124	0.122	0.115	0.107	0.094	0.089
			8	0.093	0.091	0.086	0.080	0.070	0.065
			10	0.074	0.073	0.068	0.063	0.055	0.052
			12	0.062	0.061	0.057	0.053	0.046	0.043
					U.S.A.				
Market index	0.0630	0.1776	**2**	0.996	0.986	0.950	0.908	0.835	0.803
Short-term interest	0.0159		**4**	0.498	0.491	0.464	0.435	0.386	0.366
			5	0.398	0.392	0.370	0.345	0.305	0.288
			6	0.332	0.327	0.307	0.286	0.251	0.237
			8	0.249	0.245	0.230	0.213	0.186	0.175
			10	0.199	0.196	0.183	0.170	0.148	0.139
			12	0.166	0.163	0.153	0.141	0.123	0.115

Note: Mean and Std.Dev. of continuously compounded rates from 1968:Q1 to 2019:Q4 (*208* quarterly observations). The proportion of the wealth allocated to equities is given by Equation (16).

23 From Afonso (2014), we compute the average participation of stocks in the total financial assets of Households as the ratio of the sum of equities and equity funds to the difference between total household wealth and fixed assets, i.e., (Equity + Equity Fund)/(Total Household Wealth - Fixed Assets).

Strikingly, given the parsimony of Merton's (1969) model, those two allocations can result from a risk aversion $\gamma = 5$ and investment horizons between 10 to 20 years in each country. Small perturbations to the equity premium, or to the volatility, make $\gamma = 4$ or $\gamma = 6$ also possible, as well as horizons of from 5 to 25 years. [24]

Although results from such stylized model carry the caveats raised in the literature, they are still valuable disciplined guesses and deserve further analysis. Is a relative risk aversion of $\gamma \approx 5$ plausible? Is the investment horizon between 10 to 20 years representative?

From survey responses to hypothetical situations, Kimball et al. (2008) estimate the distribution of the relative risk aversion (through its reciprocal, i.e., the relative risk tolerance $1/\gamma$). Its 0.25 , 0.50 and 0.75 fractiles are respectively 3.9 , *6.3* and *10.3* , with mean equal to *8.2* and mode equal to *3.7* , evincing that $\gamma \approx 5$ is in the center of the distribution.

Regarding the average investment horizon, although we are not aware of data for the average duration of household equity investments, these bounds sound plausible, given the "planning horizon" figures in the Survey of Consumers Finance by the Federal Reserve Board. It asks survey respondents about their most important saving and planning horizons. As described in Hong and Hanna (2014), *65.7%* of the respondents report planning horizons longer than one year, and *14.3%* report horizons longer than ten years.

[24] For an example, see the Appendix exercise when the short-term interest rate $\bar{r}_f$ is estimated and the covariance between equity premium and interest rate is negative (though small).

4. Conclusions

In this paper, we tell the history of stock market returns in Brazil during the 1968-2019 period. In addition to documenting historical Brazilian long-term equity returns and premium, we appraise them in comparison with U.S. data and through the lens of Merton's (1969) model, providing insights on asset allocation in emerging economies.

Through various descriptive statistics of the sample returns, including higher moments and the Aumann and Serrano (2008) riskiness index, we indicate that the most striking difference between the Brazilian and the U.S. stock markets is the enormous variance of the former.

Following Jacquier et al. (2003), we assess the relative biases of arithmetic and geometric methods in these two countries with very different volatilities and compute an unbiased expected return estimator that penalizes longer-horizon returns for higher volatility due to the increasing imprecision of estimates. Because the Brazilian stock market is very volatile, its expected return point estimates decrease considerably with the investment horizon.

Most interesting from an asset pricing research perspective, we show that the different Brazilian and U.S. stock market returns can result from the demands of investors that handle risk similarly. In Merton's (1969) optimal long-term allocation model, we show that the much higher Brazilian equity-premium volatility discourages heavier investments in stocks, despite expected returns being higher in Brazil than in the U.S.. With similar risk aversions, Brazilians should invest less in stocks than North Americans.

In sum, our results are consistent with an equilibrium of emerging financial markets where the demand for equities is low, despite stocks issued at a high cost of equity, because of the perceived risk. National investors can be modeled alike, in spite of the differences in macroeconomic environments.

References

Afonso, J. R. (2014) Income Tax and Distribution of Income and Wealth: The Fiscal Statistics and Pressing Debate in Brazil, *Revista da Receita Federal* 1, 28-60.

Aumann, R. and Serrano, R. (2008) An Economic Index of Riskiness, *Journal of Political Economy* 116, 810-836.

Barberis, N. (2000) Investing in the Long Run When Returns are Predictable, *Journal of Finance* 55, 225-264.

Bonomo, M. and Garcia, R. (2001) Tests of conditional asset pricing models in the Brazilian stock market, *Journal of International Money and Finance* 20, 71-90.

Campbell, J. (2003) Consumption-Based Asset Pricing, in: G. Constantinides, M. Harris and R. Stulz (eds.), *Handbook of the Economics of Finance*, Elsevier Science, pp. 801-885.

Campbell, J., and Thompson, S. (2008) Predictive Excess Stock Returns Out of Sample: Can Anything Beat the Historical Average? *Review of Financial Studies* 21, 1509-1531.

Cont, R. (2001) Empirical properties of asset returns: Stylized facts and statistical issues, *Quantitative Finance* 1, 223-236.

Cysne, R. (2006) Equity Premium Puzzle: Evidence from Brazilian Data, *Economia Aplicada* 10, 161-180.

Dimson, E., Marsh, P. and Staunton, M. (2008) The World Equity Premium: a smaller puzzle, in R. Mehra (ed.), *Handbook of the Equity Risk Premium*, Elsevier, Amsterdam, pp. 467-514.

Dittmar, R. (2002) Nonlinear Pricing Kernels, Kurtosis Preference, and Evidence from the Cross Section of Equity Returns, *Journal of Finance* 57, 369-403.

Fama, E. and French K. (1998) Value versus Growth: The International Evidence, *Journal of Finance* 53, 1975-1999.

Fama, E. and French, K. (2002) The Equity Premium, *Journal of Finance* 57, 637-659.

Fama, E. and French, K. (2018a) Volatility Lessons, *Financial Analyst Journal* 74, 42-53.

Fama, E. and French, K. (2018b) Long-Horizon Returns, *Review of Asset Pricing,* 8, 232-252.

Goetzmann, W. and Ibbotson, R. (2008) History and the Equity Risk Premium, in: R. Mehra (ed.), *Handbook of the Equity Risk Premium*, Elsevier, Amsterdam, pp. 515-529.

Harvey, C. and Siddique, A. (2000). Conditional skewness in asset pricing tests, *Journal of Finance* 55, 1263-1295.

Homm, U. and Pigorsch, C. (2012) Beyond the Sharpe Ratio: an application of the Aumann-Serrano index to performance measurement, *Journal of Banking & Finance* 36, 2274-2284.

Hong, E. and Hanna, S. (2014) Financial Planning Horizon: A Measure of Time Preference or a Situational Factor? *Journal of Financial Counseling and Planning* 25, 174-196.

Hughson, E., Stutzer, M. and Yung, C. (2006) The Misuse of Expected Returns, *Financial Analyst Journal* 62, 88-96.

Jacquier, E., Kane, A. and Marcus, A. (2003) Geometric or Arithmetic Mean: a reconsideration, *Financial Analyst Journal* 59, 46-53.

Jacquier, E., A. Kane and A. Marcus (2005) Optimal Estimation of the Risk Premium for the Long Run and Asset Allocation: a case of compounded estimation risk, *Journal of Financial Econometrics* 3, 37-55.

Jagannathan, R., McGrattan, E., Scherbina, A. (2000) The declining U.S. equity premium, *Quarterly Review* Fall, 3-19.

Kan R., Zhou, G. (2009) What Will the Likely Range of My Wealth Be? *Financial Analysts Journal* 65, 68-77.

Kimball, M., Sahm, C. and Shapiro, M. (2008) Imputing Risk Tolerance from Survey Responses, *Journal of the American Statistical Association* 103, 1028-1038.

Mehra, R. and Prescott, E. (1985) The Equity Premium: A Puzzle, *Journal of Monetary Economics* 15, 145-161.

Mehra, R. and Prescott, E. (2003) The Equity Premium in Retrospect, in: G. Constantinides, M. Harris and R. Stulz (eds.), *Handbook of the Economics of Finance*, Elsevier, Amsterdam, pp. 887-936.

Mehra, R. and Prescott, E. (2008) The Equity Premium: ABCs, in: R. Mehra (ed.), *Handbook of the Equity Risk Premium*, Elsevier, Amsterdam, pp. 1-36.

Merton, R. (1969) Lifetime Portfolio Selection Under Uncertainty: the continuous time case, *Review of Economics and Statistics* 51, 247-257.

Rouwenhorst, K. (1999) Local Return Factors and Turnover in Emerging Stock Markets, *Journal of Finance* 54, 1439–1464.

Varga, G. and Brito, R. (2016) The Cross-Section of Expected Stock Returns in Brazil, *Brazilian Review of Finance* 14, 151–187.

Welch, I. and Goyal, A. (2008) A Comprehensive Look at the Empirical Performance of Equity Premium Prediction, *Review of Financial Studies* 21, 1455-1508.

Appendix

When the expected equity return $\bar{r}_e$ and short-term interest $\bar{r}_f$ have to be estimated, the portfolio value H periods into the future is lognormal with parameters:

$$ln(V_H) \sim N(\alpha_H, \sigma_H^2) \equiv N\left(\left\{\begin{matrix}\left[\bar{r}_f + w(\bar{r}_e - \bar{r}_f)\right] \\ -\frac{1}{2}\left[\sigma_f^2 + 2w\sigma_{f,e-f} + w^2\sigma_{e-f}^2\right]\end{matrix}\right\} H, \left[\sigma_f^2 + 2w\sigma_{f,e-f} + w^2\sigma_{e-f}^2\right] H\right).$$

Because we admit that there is a real short-term interest risk and do not know $\bar{r}_e$ and $\bar{r}_f$, which we have to estimate with the dataset $\mathcal{D}$, the optimal allocation is:

$$w^* = \frac{\left(\hat{r}_e - \hat{r}_f + \frac{1}{2}\sigma_{e-f}^2\right) - \sigma_{f,e-f}\left[\gamma + \frac{H}{T}(\gamma - 1)\right]}{\sigma_{e-f}^2\left[\gamma + \frac{H}{T}(\gamma - 1)\right]}. \qquad (A.1)$$

Note that (A.1) incorporates the sample variation of $\hat{r}_f$. Additionally, the covariance between short-term interest rate and equity premium $\sigma_{f,e-f} = \left(\sigma_{f,e} - \sigma_f^2\right)$ in the numerator takes advantage of diversification opportunities. Smaller $\sigma_{f,e}$ and greater σ_f^2 justify heavier allocations in equities. Regarding σ_{e-f}^2 in the denominator of Equation (A.1), recall that: $\sigma_{e-f}^2 = \left(\sigma_e^2 - 2\sigma_{f,e} + \sigma_f^2\right)$.

Table A - Weights allocated to equity for different horizons according to Equation (A.1)

	Mean	Std. Dev.	Covar.	γ	With known parameters	Horizon (H in years)				
						1	5	10	20	25
					Brazil					
Market index	0.0677			**2**	0.40	0.39	0.38	0.36	0.34	0.32
Short-term int.	0.0086	0.0719		**4**	0.21	0.21	0.20	0.19	0.17	0.16
Equity premium		0.5009		**5**	0.17	0.17	0.16	0.16	0.14	0.13
E.prem.xS.-t. int.			-6.9E-03	**6**	0.15	0.15	0.14	0.13	0.12	0.12
				8	0.12	0.12	0.11	0.11	0.10	0.09
				10	0.101	0.100	0.095	0.090	0.082	0.079
				12	0.089	0.088	0.084	0.080	0.073	0.070
					U.S.A.					
Market index	0.0630			**2**	1.04	1.03	0.99	0.95	0.87	0.84
Short-term int.	0.0159	0.0409		**4**	0.52	0.51	0.48	0.45	0.40	0.38
Equity premium		0.1730		**5**	0.42	0.41	0.39	0.36	0.32	0.30
E.prem.xS.-t. int.			-2.2E-05	**6**	0.35	0.34	0.32	0.30	0.26	0.25
				8	0.26	0.26	0.24	0.22	0.19	0.18
				10	0.208	0.204	0.191	0.177	0.155	0.145
				12	0.173	0.170	0.159	0.148	0.128	0.121

Note: Mean and Std. Dev. of continuously compounded rates from 1968:Q1 to 2019:Q4 (*208* quarterly observations). The proportion of the wealth allocated to equities is given by Equation (A.1).