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Motivated to some extent by the empirical significance of student loans in the U.S., this paper incorporates knowledge capital formation by working households financed through debt to a demand-led dynamic model of physical and knowledge capital utilization and output growth. Average labor productivity varies positively with the average knowledge capital across the labor force. A rise in labor productivity resulting from knowledge capital accumulation is fully passed on to the real wage, so that the wage share remains constant. In the unique long-run equilibrium, which is stable, an exogenous rise in the wage share raises the rates of physical capital utilization and output growth but has an ambiguous effect on the rate of employment (which also measures the rate of knowledge capital utilization). The long-run equilibrium also features the following interrelated results: the output growth rate is greater than the exogenous interest rate; the debt ratio (working households' debt as a ratio of either the physical or the knowledge capital, or the output) is independent from the interest rate; and the allocation of a higher (lower) proportion of wage income to debt repayment (consumption) raises instead of lowers the debt ratio, which we dub the paradox of debt repayment.

Keywords: Household debt; knowledge capital; capacity utilization; employment rate; output growth.

JEL Codes: E12, E22, E24.

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1. Introduction

The role of human capital accumulation as a source of economic growth has been extensively explored in mainstream growth theory. In an early contribution, Lucas (1988), building upon Uzawa (1965), assumes that individuals choose periodically how to allocate their non-leisure time between current production and skill acquisition (or schooling), where the latter raises productivity in future periods. Human capital accumulation, by involving constant returns to the existing stock of human capital, arises as a source of sustained long-run growth. Mankiw, Romer and Weil (1992) incorporate accumulable human capital, along with physical capital and labor, as an additional production factor into an otherwise standard Solow model. As a result, the level of output per worker varies positively with both the level of physical capital per worker and the level of human capital per worker. Analogously to the accumulation of physical capital, in Mankiw, Romer and Weil (1992) the rate of human capital accumulation is fully and automatically determined by the availability of savings brought about by foregone consumption by individuals.

Admittedly, such mainstream growth approach, by assuming that the economy always grows at full capacity utilization, does neglect both the role of aggregate effective demand in growth dynamics and the impact of autonomous investment in human capital formation on aggregate effective demand. Meanwhile, demand-driven approaches to growth dynamics have relegated any closer attention to human capital formation through education (and to 'knowledge' capital accumulation more broadly) as narrowly supply-sided. One notable exception is Dutt (2010), who formalizes the process of skill acquisition in a neo-Kaleckian framework so that both the number of high-skilled and low-skilled workers and their respective wages vary over time and impact upon the interaction between income distribution and economic growth. Relatedly, Lima, Carvalho and Serra (2021) embed accumulation of human capital through the provision of universal public education by a balanced-budget government in a neo-Kaleckian dynamic model of capital utilization, income distribution and economic growth. The level of education, as represented by the stock of human capital, positively affects both workers' productivity in output production and (partly also as a result of it) their bargaining power in the labor market.

As the making of costly investments in human capital formation through education is one of the main types of productivity-enhancing knowledge accumulation, this paper explores several macroeconomic implications of the accumulation of knowledge capital by working households as mediated by its expansionary effect on labor productivity in a demand-led dynamic model of (physical and knowledge) capital capacity utilization and output growth. The model features knowledge capital accumulation by working households financed through debt as a further source of aggregate effective demand along with expenditures in investment in physical capital and consumption. Analogously to the determination of desired investment in physical capital by firms as independent from saving out of current income, the model features an independent investment function to describe the dynamics of knowledge capital accumulation by working households. This independence of the deliberate accumulation of knowledge capital by working households is accommodated by an endogenous supply of credit money, which accounts for the debt-financed nature of the desired investment in such another accumulable capital asset. Given that the aggregate stock of knowledge capital remains uniformly distributed in the labor force, which is always in excess supply, unemployed labor also means unutilized knowledge capital. As a result, the economy operates with excess productive capacity not only in physical capital and labor, as in most demand-driven growth models, but in knowledge capital as well.

Even though our model is not intended to describe specifically debt-financed knowledge capital accumulation through student loans, the recent U.S. experience with student debt is arguably illustrative of the significance of working households' financing of human capital accumulation through debt and how this is a possible route to their financial fragility. Figure 1 shows that U.S. student debt reached the considerable level of US$1.58 trillion in the first quarter of 2021, which is higher than household debt with credit card and auto loans and only lower (and substantially lower) than with mortgage. Meanwhile, aggregate household debt balances stood at US$14.64 trillion in the first quarter of 2021 (FRBNY, 2021c). As a negative sign for the longer-term sustainability of student debt, the fraction of borrowers with repayment troubles had been showing an increasing trend in the decade and a half before 2019, there existing evidence that many borrowers had only managed their repayment due to personal savings or family support (Lochner and Monge-Naranjo, 2016). In fact, 10.8% of aggregate student loan debt was 90+ days delinquent or in default in the first quarter of 2020 and transition rates into delinquency remained high (FRBNY, 2020). Given that the methodology to calculate delinquency rates on student loans does not consider those students who are out of the repayment cycle, such results on default rates tend to underestimate effective delinquency rates (FRBNY, 2021a).[1] Delinquency rates dropped considerably over the last three quarters of 2020, and about 6.5% of aggregate student debt was 90+ days delinquent or in default in the fourth quarter of 2020 (FRBNY, 2021a). The reason is that, in response to the COVID-19 pandemic, in March 2020 the U.S. Department of Education (ED) office of Federal Student Aid started providing temporary relief on ED-owned federal student loans, including suspension of loan payments, stopped collections on defaulted loans, and a 0% interest rate. In January 2021 these COVID-19 emergency relief measures were extended on ED-owned federal student loans through September 2021. In December 2021 such relief measures were again extended on ED-owned federal student loans now through May 2022. And in August 2022 the U.S. Department of Education announced a final extension of the pause on student loan repayment, interest, and collections through December 31, 2022.

In this paper, we assume that labor is always available to firms at a given real wage that makes the productivity of labor consistent with given profit margins. Accordingly, we assume that the state of the wage bargaining process is such that any increase in labor productivity which results from knowledge capital accumulation is fully passed on to the real wage, so that the wage share

[1] As noted in FRBNY (2021a), about half of the outstanding student loan debt was then in deferment, in grace periods or in forbearance and therefore temporarily not in the repayment cycle. Thus, among loans in the repayment cycle delinquency rates are roughly twice as high.

in income remains constant over time. This can be intuitively seen as resulting from a steady state of relative bargaining power of workers and capitalists that does not grant to any of these parties the possibility of raising its share in income. Therefore, while insufficient aggregate effective demand causes the aggregate stock of knowledge capital to be underutilized, it turns out that employed workers receive a full wage return on their knowledge capital. In fact, in our model economy the wage share in income, which is computed as the ratio of real wage to labor productivity, gives a simplified measure of the wage return on knowledge capital. As any increase in labor productivity brought about by knowledge capital accumulation is fully and immediately passed on to the real wage, employed workers are able to collect the full wage return on their stock of knowledge capital, so that the functional distribution of income remains constant.

Admittedly, it is not always the case that labor productivity gains are fully translated into higher wages, also because the relative bargaining power of workers and capitalists is liable to change over time. In fact, although debt dynamics and workers' borrowing behavior play an important role in the labor market by influencing workers' bargaining power (Kim, Lima and Setterfield, 2019), we are abstracting from such an additional relationship.[2] As in any modelling exercise, we confine attention to a specific and manageable set of relationships, and in our case, we focus on some relationships involved in the joint accumulation of knowledge capital and debt by workers. Thus, our modelling strategy of focusing on and confining attention to a few channels and mechanisms involved in the joint accumulation of knowledge capital and debt by workers to derive definite and rationalizable analytical results is analogous to the requisite procedure of

[2] As mentioned earlier, the model set forth in Lima, Carvalho and Serra (2021) incorporates human capital accumulation through provision of universal public education by a balanced-budget government to a demand-driven analytical framework of functional distribution and growth of income. In the model, human capital accumulation positively impacts on labor productivity and workers' bargaining power in wage negotiations. Incidentally, our more inclusive research on the coupled dynamics of human capital formation, working households' debt accumulation and varying relative bargaining power of workers and capitalists in a demand-led model is already under way.

controlling for several other covariates when testing for causal relationship in an econometric estimation.

In the unique long-run equilibrium, which is stable, an exogenous rise in the wage share raises the rates of physical capital utilization and output growth but has an ambiguous effect on the rate of employment (which also measures the rate of knowledge capital utilization). In addition, the long-run equilibrium configuration features the following interrelated results also bearing important theoretical and empirical implications: the output growth rate is strictly greater than the interest rate, which is exogenously given; the debt ratio (working households' debt as a ratio of either physical or knowledge capital, or output) does not depend on the interest rate; and working households' allocation of a higher proportion of wage income to debt repayment at the expense of consumption raises instead of lowers the debt ratio, which we dub the paradox of debt repayment.

The sequence of the paper is structured in the following way. Section 2 lays out the model structure. Section 3 solves for the short-run equilibrium values of the rates of physical and knowledge capital utilization, assuming that the existing stocks of debt (and the respective flow of debt service) and physical and knowledge capital are all given. Section 4 introduces long-run issues by following the dynamics of the ratios of physical capital to knowledge capital and working households' debt to physical capital, which allows the exploration of how the rates of capital capacity utilization and output growth respond to parametric changes in the long-run equilibrium, which is unique and stable. Finally, Section 5 concludes the paper.

2. The structure of the model

The model herein will deal with a closed economy that produces a single good/service for both investment (on physical and knowledge capital) and consumption. Two homogeneous factors of production are used in the production process, physical capital and labor, and the stock of knowledge capital is assumed to remain uniformly embodied in the available labor force. These production inputs are combined through a fixed-coefficient technology:

$$X = \min\left[Kv, La(h)\right], \qquad (1)$$

where X is the output level, K is the stock of physical capital, L is the employment level, v is the ratio of capacity ouput to physical capital, $h = H / N$ is the knowledge capital stock to labor force ratio (or average knowledge capital) and $a(h)$ is the output to labor ratio (or labor productivity), which varies endogenously with the average knowledge capital. For simplicity, the technical coefficient v will be normalized to a constant equal to one. In the production function in (1), we also assume that $a(0) = 0$, $a'(h) > 0$ and $a''(h) \leq 0$. Given that we are dealing with a single good/service economy, the 'production' of knowledge capital does not constitute another production process or even productive sector. It is assumed herein that the single good/service that can be used for both physical capital accumulation and consumption can be used for knowledge capital accumulation as well. Moreover, given that unemployed workers are as skilled (or knowledge capital endowed) as employed ones, the rate of labor employment, which is determined by aggregate effective demand, measures the degree of knowledge capital utilization. Though we consider only the case in which aggregate effective demand is insufficient to yield full utilization of the existing productive capacity (in either capital or labor) at the ongoing price and wage rate, we abstract from (physical and knowledge) capital depreciation and labor deskilling. Moreover, the model is cast in real terms.

The economy is composed of two classes of households, capitalists and workers, who earn profits and wages, respectively. The functional division of aggregate income is then given by:

$$X = VL + rK, \qquad (2)$$

where V is wage rate and r is the rate of profit on physical capital, which is the flow of profits, R, as a proportion of the physical capital stock. From (1) and (2), the share of labor in income, σ, is given by:

$$\sigma = \frac{V}{a(h)}. \qquad (3)$$

To keep sharp focus on the issue of the possibly better employment prospects opened up by knowledge capital accumulation, we assume that any increase in labor productivity resulting from knowledge capital accumulation is fully and immediately passed on to the wage rate. Thus, while insufficient aggregate effective demand causes the existing aggregate stock of knowledge capital to be underutilized, employed workers receive a full wage return on their knowledge capital. This can be interpreted as resulting from a continuous state of relative bargaining power of workers and capitalists that does not allow any of these parties to raise its share in aggregate income. In effect, the wage share in income, as specified in (3), provides a simplified measure of the wage return on knowledge capital. Given that any increase in the productivity of labor brought about by knowledge capital accumulation is fully and immediately passed on to the wage rate, with the result that employed workers are always able to collect the full wage return on their accumulated knowledge capital, the wage share (and hence the distribution of aggregate income between wages and profits) remains unchanged.

Firms produce (and hire labor) according to aggregate effective demand. As we model only the situation in which excess productive capacity (in labor and capital in general) prevails, labor employment is determined by production:

$$L = \frac{X}{a(h)} . \tag{4}$$

At a point in time, the existing technological conditions are given, having resulted from previous knowledge and physical capital accumulation. Over time, however, knowledge capital accumulation takes place as described later, which results in labor productivity growing at the proportionate rate $\hat{a}$. Formally:

$$\hat{a} = \rho(\hat{h}) , \tag{5}$$

where $\hat{h}$ is the proportionate growth rate of the knowledge capital stock to labor force ratio, with $\rho(0) \geq 0$, $\rho'(.) > 0$ and $\rho''(.) \leq 0$.

The employment rate, $e = L / N$, is linked to the state of the market for goods/services in the following way:

$$e = \frac{L}{X}\frac{X}{K}\frac{K}{N} = uk\,, \tag{6}$$

where $u = X / K$ is the rate of physical capital utilization and k stands for the ratio of physical capital stock to labor force in productivity units, that is, $k = K / (Na(h))$. This formal link between u and e resulting from the fixed-coefficient nature of the technology implies that a rise in output in the short run, when k is given, is necessarily accompanied by an increase in employment. Moreover, as the aggregate knowledge capital stock is uniformly distributed in the labor force, the employment rate also measures the rate of utilization of the aggregate knowledge capital stock. For simplicity and focus, we treat the labor force, N, as constant (but nonetheless always in excess supply) and normalize it to one, and assume that the level of labor productivity has a one-to-one correspondence with the average stock of knowledge capital, h, so that $a(h) = h = H$ and, hence $k = K / H$. Moreover, it follows that $\hat{a} = \hat{h} = \hat{H}$.

Firms' decisions regarding accumulation of physical capital are made independently from any prior savings. The implied desired growth rate of the stock of physical capital, g_K, assuming no depreciation, is given by:

$$g_K = \frac{I_K}{K} = \beta u\,, \tag{7}$$

where I_K denotes firms' desired investment in physical capital, whereas β is a strictly positive parameter capturing accelerator-type effects.

Analogously to the determination of investment in physical capital as independent from prior savings, working households' decisions to accumulate knowledge capital are also assumed to be so independent. This independence of knowledge capital accumulation is accommodated by an endogenous supply of credit money, which ensures the debt-financed nature of the desired investment in such another accumulable capital asset. We assume that working households' desired level of investment in knowledge capital is given by:

$$I_H = \gamma VL\,, \tag{8}$$

where γ is a strictly positive parameter. Analogously to specifications of the desired investment in physical capital in the Cambridge U.K. tradition, which typically have total profits (or the expectation of it) as a positive determinant, the desired level of investment in knowledge capital in (8) varies positively with the wage bill (or the expectation of it, which is then conventionally proxied by its current value). The implied desired growth rate of the stock of knowledge capital, g_H, assuming no depreciation, is given by:

$$g_H = \frac{I_H}{H} = \gamma \frac{VL}{X}\frac{X}{K}\frac{K}{H} = \gamma\sigma uk = \gamma\sigma e\,, \tag{9}$$

where e is given by (6). Recall that the knowledge capital stock is uniformly distributed in the labor force, so that the employment rate also measures the degree of utilization of such a stock. Therefore, the specification in (9) can be interpreted as also incorporating an accelerator effect, but applied to the investment in knowledge capital instead of physical capital. Interestingly, the strength of such an accelerator effect varies positively with the wage share σ. Meanwhile, for future reference, working households' desired level of investment in knowledge capital as a proportion of the physical capital stock is given by:

$$\frac{I_H}{K} = \gamma \frac{VL}{X}\frac{X}{K} = \gamma\sigma u\,. \tag{10}$$

Following the tradition of Kalecki (1971), Kaldor (1956), Robinson (1962) and Pasinetti (1962), we assume that workers and capitalists have different consumption behavior. Workers provide labor and earn wage income, of whose net value (recall that workers have to meet debt servicing obligations, as described later), which is nonetheless always strictly positive, nothing is saved. Although we also assume that the size of the labor force, N, is a constant normalized to one, the labor force (in natural and productivity units) is always in excess supply. Capitalist households receive not only profit income, which is the entire surplus over the wage bill, but also interest income as recipients of workers' debt service, and spend in consumption a constant proportion of it, $0 < 1 - s < 1$, where s denotes capitalist households' saving rate.

3. Short-run equilibrium

The short-run is defined as the time period along which the stock of physical capital, K, the stock of knowledge capital, H, the output-labor ratio, a, the wage rate, V (and thus the wage share, σ) as well as the debt variables to be describe shortly, can all be taken as given. Supply-demand equilibrium in the market for goods/services is given by:

$$X = C_w + C_c + I_H + I_K, \tag{11}$$

where C_w and C_c represent aggregate consumption by working households and capitalist households, respectively. Thus, knowledge capital accumulation by working households is an extra source of aggregate effective demand alongside with investment expenditures in physical capital by firms and consumption by the two classes of households. For future reference, the equilibrium condition in (11) as a proportion of the physical capital stock is given by:

$$u = \frac{C_w}{K} + \frac{C_c}{K} + \frac{I_H}{K} + g_K. \tag{12}$$

We assume that the debt servicing by working households follows an income-driven repayment plan bearing some similarity to repayment plans applicable to federal student loan payments in the U.S.[3] More precisely, working households' consumption is given by:

[3] The main safety net available to U.S. borrowers of federal student loans facing excessive monthly payments is the income-driven repayment. The Income-Based Repayment Plan (IBR), available since 2009, is the most widely available such repayment plan for federal student loans. Monthly payments are 10% or 15% of discretionary income, and are recalculated each year based on the updated income and family size. Any outstanding balance will be forgiven if not repaid in full after 20 or 25 years. In August 2022 the U.S. Department of Education proposed a rule to create a new income-driven repayment plan.

$$C_w = \phi VL\,, \tag{13}$$

where $0 < \phi < 1$ is a parameter. We denote by $0 < 1 - \phi < 1$ the proportion of the wage income that working households divert from consumption for the purpose of debt repayment, to which we refer simply as coefficient of debt repayment. Thus, the change in the stock of debt held by working households is given by:

$$\dot{D} = iD - (1-\phi)VL + I_H\,, \tag{14}$$

where D is the stock of debt in real terms held by working households, so that $\dot{D} = dD/dt$ is the change in such a stock, and $i > 0$ denotes the interest rate. The stock of debt held by working households is given in the short run, but varies over time as described shortly, while the interest rate, for choice of focus, is assumed to remain constant throughout. Therefore, given that the stock of debt, D, and the real wage, V, vary over time, while the employment level, L, is an adjusting variable in the short run, as described shortly, the proportion of the wage income that working households divert from consumption for the purpose of debt repayment, $(1-\phi)VL$, may be greater than, equal to or lower than the interest payment due, iD, in a given short run. It is reasonable that the amount of debt repayment is the adjusting variable when the proportion of the wage income that working households divert from consumption for the purpose of debt repayment is not enough to even cover the interest payment. This way the debt service does not happen to compromise a justifiable minimum level of consumption by working households. In effect, this appears to be (or is compatible with) the social and economic logic underlying the income-driven repayment plans applicable to federal student loan payments in the U.S. on

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The proposed rule would reduce by up to 50% the amount that borrowers have to pay each month and would increase the amount of income that is considered nondiscretionary income and hence protected from repayment. This rule would also forgive student loan balances after 10 years of payments, in lieu of the present 20 years under several income-driven repayment plans, for borrowers with original loan balances amounting $12,000 or less. Mueller and Yannelis (2019) offer evidence that the IBR program has been successful at reducing student loan defaults.

which our specification in (13) is partially based. Therefore, when $(1-\phi)VL = iD$, there is no repayment of the principal and all the increase in the stock of debt is equal to the flow of investment in knowledge capital, I_H. When $(1-\phi)VL > iD$, there is some repayment of the principal and whether the stock of debt will increase, remain the same or fall depends on whether the excess of $(1-\phi)VL$ over iD is lower than, equal to or greater than I_H, respectively. And when $(1-\phi)VL < iD$, there is no repayment of the principal and the increase in the stock of debt is equal to I_H plus the unpaid interest cost added to the outstanding principal.

Using (8), we can re-write (14) as:

$$\dot{D} = iD + (\phi + \gamma - 1)VL, \qquad (14\text{-a})$$

where we assume that $\phi + \gamma > 1$, so that there is no net debt repayment and the stock of debt always varies positively no matter what the economically relevant values of D, V, and L are. As will be seen in the next section, however troublesome such debt dynamics may seem, it does not bring about an explosive behavior of the debt ratio in the long run, be the debt ratio defined as the debt as a proportion of physical capital, knowledge capital or aggregate output.

Given our previously made assumption that capitalist households allocate to consumption a constant proportion, s, of their profit and interest income, it follows that:

$$C_c = (1-s)(R + iD). \qquad (15)$$

We follow Dutt (2006) and Kapeller and Schütz (2015) in assuming that the debt service increases the purchasing power of profit recipients who lend in an amount proportional to the interest on the debt, and that profit recipients do not curtail their consumption when they lend. As argued in Dutt (2006), the reason is that while lending reduces the flow of cash available for consumption by profit recipients, it also increases their assets.

Given that aggregate output is determined by aggregate effective demand, and labor (along with the knowledge capital uniformly embodied in the labor force) is always in excess supply at the ongoing wage rate, the rate of physical capital utilization, u, adjusts for the equilibrium in the market for goods/services in (11) to obtain. By normalizing (13) and (15) by the physical capital stock and substituting the resulting expressions (along with (7) and (10)) into the normalized goods/services-market equilibrium condition in (12), we obtain:

$$u = \left(\frac{\phi VL}{X}\frac{X}{K}\right) + (1-s)\left(\frac{R}{K} + \frac{iD}{K}\right) + \gamma\sigma u + \beta u \,. \qquad (16)$$

Using (2) and (3) (along with the previously made assumption that $a(h) = h = H$) to re-write the profit rate on physical capital, R/K, in terms of the profit share, $1-\sigma$, and the physical capital utilization as $R/K = (R/X)(X/K) = (1-\sigma)u$, and then substituting the resulting expression into (16), we can solve for the short-run equilibrium physical capital utilization to obtain that:

$$u = \frac{(1-s)i\delta}{\Omega}, \qquad (17)$$

where $\delta = D/K$ denotes the debt ratio and $\Omega = s(1-\sigma) - (\phi + \gamma - 1)\sigma - \beta$. To ensure that the demand-led output-adjustment stability condition known as the Keynesian stability condition is satisfied, we further assume that saving as a proportion of the physical capital stock, which is given by $u - (C_w/K) - (C_c/K)$ is more responsive to changes in physical capital utilization than investment (in both physical and knowledge capital) as a proportion of the physical capital stock, which is given by $(I_H/K) + g_K$. It can be checked that this condition is equivalent to a positive denominator in (17). Intuitively, given the functional distribution of income and the saving propensity of capitalist households, the Keynesian stability condition is more easily satisfied, the lower the values of ϕ, γ, and β are, for these are parameters positively affecting the magnitude of effective demand injections. Note that $1-\phi$ denotes workers' propensity to save, which is nevertheless immediately converted into their coefficient of debt repayment.

Employing (6), the short-run equilibrium rate of employment, which also measures the rate of knowledge capital utilization, is given by:

$$e = \frac{(1-s)i\delta k}{\Omega}. \tag{18}$$

As routinely assumed in one-good macroeconomic models featuring physical capital and labor as factors of production, we have made the conveniently simplifying assumption that the single good/service produced in the economy can be alternatively used for consumption or physical capital accumulation purposes. To add further convenience and tractability, we assume that such single good/service can also be used for knowledge capital accumulation. In the long-run equilibrium, therefore, the growth rate of output, g^*, can be measured by the growth rate of either kind of capital, given that both physical and knowledge capital grow at the same rate in the long-run equilibrium:

$$g^* = g_K^* = g_H^*. \tag{19}$$

By virtue of the demand-led nature of the model, the short-run equilibrium rates of physical capital utilization and employment in (17) and (18), respectively, vary positively with the parameters ϕ, γ, and β, and negatively with the capitalists' saving rate. Yet such endogenous variables vary positively with each of the separate variables which compose the debt service as a proportion of the physical capital stock, which are the interest rate and the debt ratio (recall that the latter is given in the short run, whereas the former remains constant throughout). The intuition is that, per (13)-(15), for example, an increase in the flow of debt service received (and partially spent on consumption) by capitalist households represents a credit-fuelled increase in aggregate effective demand, given that working households meet such an increase in their flow of debt obligations entirely by taking on new debt. Meanwhile, the impact of an increase in the wage share (which, per (3), also measures the wage return on knowledge capital) on the short-run equilibrium value of the rate of physical capital utilization is given by:

$$\frac{\partial u}{\partial \sigma} = u_\sigma = \frac{[s + (\phi + \gamma - 1)](1 - s)i\delta}{\Omega^2} > 0 \,. \tag{20}$$

Therefore, *ceteris paribus*, a rise in the wage share, by redistributing income from capitalist households who save to working households who spend in consumption all of their net wage income, raises aggregate demand and thereby boosts the rates of physical capital utilization and employment in the short-run equilibrium (recall that there is no net debt repayment by working households and their stock of debt always varies positively in part due to the debt financing of the investment in knowledge capital, as implied by our assumption that $\phi + \gamma > 1$ in (14-a)). Also, given that overall investment demand features a double accelerator effect (one working via investment in physical capital by capitalist households, the second operating via investment in knowledge capital by working households), aggregate effective demand rises even further. Meanwhile, it follows from (17)-(18) that, *ceteris paribus*, a rise in the ratio of physical capital to knowledge capital, k, does not impact on the short-run equilibrium rate of physical capital utilization, but it raises the short-run equilibrium rate of employment (recall that the latter rate also measures the rate of knowledge capital utilization).

It remains to be properly investigated how the rates of physical capital utilization, knowledge capital utilization (or labor employment) and output growth in (17)-(19) vary with parameters and exogenous variables of the model in the long-run equilibrium. The latter, explicably, should feature stationary values of the ratios involving the three accumulable assets, namely, the ratios of physical capital to knowledge capital, k, and of debt to physical capital, δ (and therefore of debt to knowledge capital). Yet it can be shown that for any strictly positive values of the state variables k and δ, the exogenously given wage share plays a significant role in the evolution of the stock of debt held by working households. In effect, recall that our assumption in (14-a) that $\phi + \gamma > 1$ implies that there is no net debt repayment and the stock of debt always varies positively no matter what the values of D, V, and L are. But it is possible that the proportion of the wage income that working households allocate to debt repayment, $(1-\phi)VL$, is greater than, equal to or lower than the interest payment due, iD, in a given short run, with the wage share playing a significant role in the determination of the ultimate result. This can be seen by scaling the relevant expression, $(1-\phi)VL-iD$, by the physical capital stock, which results in $\Phi=(1-\phi)\sigma u-i\delta$. As established in Appendix A, for a given strictly positive value of the state variable δ, which we will show to be the case in the long-run equilibrium in the next section, $\Phi(\bar{\sigma})=0$, where such threshold value of the wage share is given by:

$$\bar{\sigma}=\frac{s-\beta}{s\phi+\gamma}. \tag{21}$$

Moreover, as $\partial\Phi/\partial\sigma>0$, it follows that $\Phi>0$ ($\Phi<0$) when $\sigma>\bar{\sigma}$ ($\sigma<\bar{\sigma}$): as a proportion of the physical capital stock, the amount of wage income that working households allocate to debt repayment instead of consumption is always greater (lower) than the interest payment due. Intuitively, in light of the demand-led nature of the model, $\bar{\sigma}$ varies negatively with β, ϕ and γ, and positively with s.

4. Long-run equilibrium

For the long run, we assume that the short-run equilibrium values of the variables are always attained, with the economy moving over time due to changes in the stocks of physical capital, K, knowledge capital, H, and working household's debt, D. Recall that we have assumed that the aggregate stock of knowledge capital remains uniformly distributed in the labor force (the measure of which we have normalized to a constant equal to one), and that the productivity of labor is equal to the average stock of knowledge capital, which together imply that the proportionate growth rates of the aggregate stock of knowledge capital and labor productivity remain one and the same. Thus, one way of following the behavior of the system over time is by investigating the dynamic behavior of the short-run state variables k, the ratio of physical capital stock to knowledge capital stock, and δ, the ratio of working households' stock of debt to physical capital stock.[4] From the definition of these two variables, we have the following state transition functions in terms of proportionate growth rates:

$$\hat{k} = \hat{K} - \hat{H} \,, \tag{22}$$

and:

$$\hat{\delta} = \hat{D} - \hat{K} \,. \tag{23}$$

Substitution of (6), (7) and (9) into (22) yields:

$$\hat{k} = (\beta - \gamma\sigma k)u \,, \tag{24}$$

where u is given by (17).

Meanwhile, (23) can be re-written as follows:

$$\hat{\delta} = \hat{D} - \hat{K} = \frac{\dot{D}}{K}\frac{K}{D} - g_K \,. \tag{25}$$

Therefore, substitution from (14-a) and (7) into (25) yields:

$$\hat{\delta} = \frac{(\phi + \gamma - 1)\sigma u}{\delta} + i - \beta u \,, \tag{26}$$

where u is likewise given by (17).

Using (24) to set $\hat{k} = 0$ and noticing from (26) and (17) that $\hat{\delta}$ does not depend on the ratio of physical to knowledge capital, the latter's (unique and strictly positive) long-run equilibrium value is given by:

[4] In a long-run equilibrium with constant ratios of physical capital to knowledge capital and working households' debt to physical capital, the ratio of working households' debt to knowledge capital is also constant. Moreover, given that in a long-run equilibrium physical capital utilization (as measured by the ratio of output to physical capital) is constant, the ratio of working households' debt to output is constant as well.

$$k^* = \frac{\beta}{\gamma\sigma}. \tag{27}$$

Intuitively, the long-run equilibrium value of the ratio of physical capital to knowledge capital varies positively with the parameter associated with the accelerator effect in the expression for the desired rate of physical capital accumulation, which is β in (7), and negatively with the parameters related to the accelerator effect in the expression for the desired rate of accumulation of knowledge capital, which feature multiplicatively as $\gamma\sigma$ in (9). It follows that the long-run equilibrium value of the ratio of physical capital to knowledge capital in (27) is equal to the ratio between the parameters associated with the accelerator effect in the respective rates of accumulation.

Using (26) and (17) in setting $\hat{\delta} = 0$, the (unique) long-run equilibrium value of the ratio of working households' debt to physical capital is given by:

$$\delta^* = \frac{s[1-(\phi+\gamma)\sigma]-\beta}{\beta(1-s)}, \tag{28}$$

which is strictly positive, as demonstrated in Appendix B (where it is also shown how the long-run equilibrium value of the debt ratio in (28) varies with each of the parameters of which it is a function). Interestingly, the long-run equilibrium value of the debt ratio varies negatively (positively) with the proportion of the wage bill allocated to consumption (debt repayment), which is given by ϕ $(1-\phi)$. Indeed, the same seemingly paradoxical result is obtained if the debt ratio is expressed as the ratio of working households' debt to knowledge capital, the long-run equilibrium value of which is represented by $d^* = (D/H)^* = k^*\delta^*$ (recall from (27) that k^* does not depend on ϕ). As aptly discussed in Lavoie (2014, ch. 1), several macroeconomic paradoxes, or fallacies of composition, have been uncovered in heterodox macroeconomics. Two famous such paradoxes are Keynes' paradox of thrift, according to which a higher saving rate leads to a reduced output, and the Kaleckian paradox of costs, according to which a higher real wage leads to a higher profit rate. Another revealing example is the paradox of debt, which is also based on the principle of effective demand and was suggested by Steindl (1952, ch. 9). The basic idea is that although an economic agent may individually suceed in reducing her debt ratio by cutting back on borrowing-financed expenditures, a large aggregate of economic agents behaving the same way may fail to achieve such a reduction and instead face an increased debt ratio. Thus, our result that the long-run equilibrium value of the debt ratio (the stock of debt of working households as a proportion of the stock of either physical or knowledge capital, or the flow of aggregate output) varies negatively with the fixed proportion of the wage bill allocated to debt repayment instead of consumption can be interpreted as a variant of the standard paradox of debt. We dub such a variant the *paradox of debt repayment*.

We should clarify that, if working households saved, and forfeited savings to increase debt servicing, such an additional debt servicing could boost overall consumption and hence demand formation, so that the alluded paradox might not hold in the long-run equilibrium. The intuitive reason is that these forfeited savings would be transferred to creditor capitalists who partially consume some of their additional interest income.[5] Suppose that workers save a proportion of the wage income and the pecking order is that consumption has a prior claim on wage income, with saving and debt servicing then sharing in a fixed proportion the income that is left. All else constant, a fall in the proportion of wage income devoted to consumption results in a rise in the proportions devoted to saving and debt serving. Although both worker saving and debt servicing rise, and hence worker consumption falls, the debt ratio rises and the paradox of debt repayment would hold. Yet if the proportion of consumption in wage income remains constant and workers relies on a reduction in the share of saving to raise the share devoted to debt servicing, the debt ratio declines and the considered paradox would disappear. Suppose again that workers save a

[5] In fact, as shown by Setterfield, Kim and Rees (2016) and Setterfield and Kim (2016), the way that debtor households service their debts (specifically, if debt servicing reduces their consumption out of wage income) does matter for the ultimate impact of debt servicing on macroeconomic performance and debt ratios. An algebraic demonstration of the several results in this regard reported in the sequence of the paragraph above is available from the authors upon request.

proportion of the wage income, but now the pecking order is that saving has a prior claim on wage income, with consumption and debt servicing sharing in a fixed proportion the income that is left. All else constant, a rise in the proportion of wage income net of saving devoted to debt servicing leads to a fall in the proportion of such wage income devoted to consumption, while the share of wage income devoted to saving remains constant. As a result, although debt servicing increases, with worker consumption falling and worker saving remaining constant, the debt ratio rises and the paradox of debt repayment would hold. Yet if workers relies on a reduction in the share of saving in wage income to raise the share devoted to debt servicing, which raises the share devoted to consumption as well, the debt ratio falls and the considered paradox would disappear. Finally, suppose again that workers save a proportion of the wage income, but now the pecking order is that debt servicing has a prior claim on wage income, with consumption and saving then sharing in a fixed proportion the income that is left. All else constant, a rise in the proportion of wage income devoted to debt servicing leads to a decline in the proportions devoted to consumption and saving. Although debt servicing increases, worker consumption and saving decrease. As a result, if workers' marginal propensity to consume out of the wage income net of debt servicing is greater (lower) than capitalist households' marginal propensity to consume out of the profit and interest income, the debt ratio rises (falls) and the paradox of debt repayment holds (does not hold). Meanwhile, if workers reduce the proportion of wage income net of debt servicing devoted to saving, while maintaining the proportion of wage income devoted to debt servicing (which means that debt servicing remains constant), the debt ratio nonetheless declines. This results from the net consumption demand injection owing to the fall in the proportion of saving in wage income.

The long-run equilibrium debt ratio in (28) varies negatively with the two parameters defining the strength of the accelerator effect in the desired rate of accumulation of knowledge capital, which are γ and σ in (9). Given that per (27) k^* as well varies negatively with the same two parameters, it follows that the debt ratio expressed as the ratio of working households' debt to knowledge capital also varies negatively with the wage share and the sensitivity of working households' desired investment in knowledge capital to the wage bill. The long-run equilibrium working households' debt (as a proportion of either the physical or the knowledge capital stock) also varies positively with the rate of saving of capitalist households (recall from (27) that k^* does not depend on s). Another result the intuition for which is related to the demand-led nature of the model is that the long-run equilibrium working households' debt (as a proportion of either the physical or the knowledge capital) varies negatively with the parameter determining the strength of the accelerator effect in the desired rate of accumulation of physical capital, which is given by β in (7). Although k^* varies positively with β, while δ^* varies negatively with it, it can be straightforwardly checked with (27)-(28) that the net effect on $d^* = k^*\delta^*$ is negative.

A final interesting result regarding the behavior of the short-run state variables k and δ in the long-run equilibrium relates to how they respond to a change in the interest rate. In fact, (27)-(28) show that the long-run equilibrium values of such state variables are *independent* from the interest rate. Therefore, the long-run equilibrium dynamic of the model features what we dub *interest rate neutrality*, in that a change in the interest rate has no impact on the long-run equilibrium values of the working households' debt as a proportion of either the physical or the knowledge capital, and the ratio of physical capital to knowledge capital. The reason, as demonstrated in (A-8)-(A-10), is that the long-run equilibrium values of the growth rate of the debt, $\hat{D}^*$, and the growth rate of physical capital, $\hat{K}^*$, respond to a change in the interest rate with a change in the same direction and of the same magnitude.

However, it should be investigated whether the unique long-run equilibrium featuring (27)-(28) is stable. Equations (24) and (26), after using (17), constitute an autonomous two-dimensional system of differential equations in which the proportionate growth rates of k and δ depend on the levels of k and δ and parameters of the system. The respective Jacobian matrix of partial derivatives, after using (17), when evaluated at the unique stationary configuration represented by (27)-(28), is the following:

$$J_{11} = \frac{\partial \hat{k}}{\partial k} = -\frac{\gamma\sigma(1-s)i\delta^*}{\Omega} < 0\,, \tag{29}$$

$$J_{12} = \frac{\partial \hat{k}}{\partial \delta} = \frac{(\beta - \gamma\sigma k^*)(1-s)i}{\Omega} = 0\,, \tag{30}$$

$$J_{21} = \frac{\partial \hat{\delta}}{\partial k} = 0\,, \tag{31}$$

$$J_{22} = \frac{\partial \hat{\delta}}{\partial \delta} = (\phi + \gamma - 1)\sigma\frac{(u_\delta \delta - u)}{(\delta)^2} - \beta u_\delta = -\beta u_\delta < 0\,. \tag{32}$$

The sign of J_{11} is unambiguously negative, given that a higher k, *ceteris paribus*, by lifting the employment rate, raises the growth rate of the stock of knowledge capital while leaving unchanged the growth rate of the stock of physical capital. Regarding the sign of J_{12}, a higher δ, *ceteris paribus*, raises both the rate of physical capital utilization (and hence the growth rate of the stock of physical capital) and the rate of employment (and hence the growth rate of the stock of knowledge capital). In the long-run equilibrium, however, given that such stocks grow at the same rate, the sign of J_{12} immediately follows, which can be confirmed by substituting (27) in (30). The reason for the sign of J_{21} is that neither working households' desired level of investment in knowledge capital as a proportion of the stock of physical capital nor the growth rate of the stock of physical capital depend on the employment rate (and hence on k). The sign of J_{22} is negative. A higher debt ratio, *ceteris paribus*, by positively affecting physical capital utilization, raises working households' investment in knowledge capital (and hence the change in debt) as a proportion of physical capital, which raises the rate of growth of debt. The same rise in the debt ratio exerts a direct downward pressure on the rate of growth of debt, though, as shown in (25). But note from (17) that the debt-ratio-elasticity of physical capital utilization is unitary, so that $u_\delta \delta - u = 0$ and the rate of growth of debt does not depend on the debt ratio.

Thus, the negative sign of J_{22} is due to the positive impact of a higher debt ratio on the rate of growth of physical capital.

Consequently, the Jacobian matrix represented by (29)-(32) features a positive determinant, $Det(J) = J_{11}J_{22} + J_{12}J_{21} > 0$, and a negative trace, $Tr(J) = J_{11} + J_{22} < 0$, so that the long-run equilibrium with $\hat{k} = \hat{\delta} = 0$, which is given by $(k, \delta) = (k^{*}, \delta^{*})$, is locally stable. In effect, as portrayed in Figure 2, this long-run equilibrium is a locally stable node. The slope of the $\hat{k} = 0$ isocline is given by $-(J_{11} / J_{12})$ and hence infinite, which explains its verticality. Since $\partial\hat{k} / \partial k$ is negative, $\hat{k}$ undergoes a steady decrease as δ increases, so that the sign of $\hat{k}$ is positive (negative) to the left (right) of the $\hat{k} = 0$ locus, which explains the direction of the horizontal arrows. Meanwhile, the slope of the $\hat{\delta} = 0$ isocline, which is given by $-(J_{21} / J_{22})$, is equal to zero. Given that $\partial\hat{\delta} / \partial\delta < 0$, it follows that $\hat{\delta}$ undergoes a steady decrease as δ increases, so that the sign of $\hat{\delta}$ is positive (negative) below (above) the $\hat{\delta} = 0$ isocline, which explains the direction of the vertical arrows.

We can then use the stable long-run equilibrium values of k^{*} and δ^{*} in (27)-(28) to evaluate how the long-run equilibrium values of the rates of physical and knowledge capital utilization and output growth vary with parametric changes. Here we only present and discuss such results, with the corresponding calculations being reserved to Appendix C.

First, recall that in the short run, which is the time span over which the pair (k, δ) is given, the equilibrium rates of physical capital utilization and employment in (17)-(18) vary negatively with the capitalists' saving rate. This paradox of thrift also applies to the long-run equilibrium value of the rate of physical capital utilization, as confirmed in (C-1), and therefore to the long-run equilibrium values of the rates of output growth and employment, in the case of the latter recalling that k^{*} does not depend on the capitalists' saving rate.

Second, a lower (higher) consumption (debt repayment) coefficient out of wage income on the part of working households ϕ $(1-\phi)$ yields lower long-run equilibrium values of physical capital utilization (per (C-2)), output growth and employment rate (in the latter case recalling that k^* does not vary with ϕ), which plays a role in its leading to a higher value of the debt ratio in the long-run equilibrium (see (B-2)). This result has the important empirical and policy implication that a more affordable debt service associated with knowledge capital accumulation results in a higher level of macroeconomic activity and a lower debt ratio.

Third, a higher wage share yields higher long-run equilibrium values of the physical capital utilization (per (C-3)) and output growth, which plays a part in its leading to a lower value of the debt ratio in the long-run equilibrium (see (B-3)). This result has the relevant empirical and policy implication that a better functional distribution of income in favor of wage-earners is effective in reducing their debt ratio. However, a higher wage share lowers the ratio of physical capital to knowledge capital in the long-run equibrium, which is equivalent to a fall in k^* in (27). Recall from (6) that the long-run equilibrium value of the employment rate (which also measures the rate of knowledge capital utilization) is given by $e^* = u^* k^*$, so that the ratio of knowledge capital utilization to physical capital utilization is inversely proportional to the ratio of physical capital to knowledge capital in the long-run equilibrium outcome. Alternatively, and intuitively, the long-run equilibrium features the equality between the stock of physical capital multiplied by its rate of utilization and the stock of knowledge capital multiplied by its rate of utilization. Therefore, with a higher wage share yielding both a higher physical capital utilization and a lower ratio of physical capital to knowledge physical capital, the resulting impact on the employment rate is ambiguous (see (C-4)).[6] An important policy implication of

[6] This result that a rise in the wage share raises output growth but has an ambiguous impact on the employment rate echoes that of demand-led models in which labor productivity growth is driven by the Kaldor-Verdoorn effect. Here a rise in the wage share increases knowledge capital formation which contributes to both aggregate demand formation (which raises aggregate output and employment) and labor productivity (which lowers aggregate employment). In demand-led models featuring the Kaldor-

this theoretical result is that ensuring workers a higher share in income may come at the cost of reducing the employment rate.

Fourth, recall from (27) that the long-run equibrium value of the ratio of physical to knowledge capital, k^*, varies negatively with the parameters associated with the accelerator effect in the desired rate of accumulation of knowledge capital, which in turn feature multiplicatively as $\gamma\sigma$ in (9). However, while a change in the wage share σ has a direct impact on aggregate demand by affecting working households' expenditures on consumption and investment in knowledge formation, a change in γ has a direct impact on aggregate demand only by affecting working households' expenditures on investment in knowledge accumulation. However, similarly to an increase in σ, as explored above, an increase in such other parameter affecting the strength of the accelerator effect in the desired rate of accumulation of knowledge capital, γ, also results in a higher long-run equilibrium physical capital utilization (per (C-2)) and output growth (per (C-6) and (C-2)), which plays a part in its leading to a lower value of the debt ratio in the long-run equilibrium (see (B-2)). But since a higher γ as well lowers the ratio of physical capital to knowledge capital in the long-run equibrium, its resulting impact on the employment rate is ambiguous (see (C-5)).

Fifth, although the impact of a rise in the parameter β measuring the strength of the accelerator effect in the desired rate of physical capital accumulation on the long-run equilibrium physical capital utilization is ambiguous (see (C-11)), the same rise has a positive impact on the long-run equilibrium growth rate of physical capital (which is also the long-run equilibrium growth rate of output) (see (C-8)-(C-9)). In efffect, the same rise has a positive impact on the long-run equilibrium employment rate (or knowledge capital utilization) as well (see (C-15)).

Verdoorn effect, aggregate demand formation increases output growth and thereby labor productivity growth through the Kaldor-Verdoorn effect, so the impact on the employment rate may be ambiguous (see, e.g., Storm and Naastepad, 2012).

Sixth, the long-run equilibrium value of the rate of growth of output is strictly greater than the interest rate, which is exogenously determined (see (C-9)). This result plays an important role in the stability of the unique long-run equilibrium configuration. Interestingly, this analytical result is an implication of our assumption that $\phi + \gamma > 1$ in (14-a), the substance of which is that there is no net debt repayment and the stock of debt of working households always varies positively.

A current policy discussion and debate for which our results are informative concerns student debt forgiveness. In fact, in late October 2022 the U.S. Department of Education released final regulations for several rules providing targeted student debt cancellation of various amounts to borrowers with federal loans. These rules would cancel up to $20,000 in debt for tens of millions of borrowers and are scheduled to go into effect July 1, 2023.[7] In addition, these rules eliminate all instances of interest capitalization (which occurs when borrowers have outstanding unpaid interest added to their principal balance) not required by statute. These rules also make it easier for borrowers working in public service to get loan forgiveness. Intuitively, our results suggest that student debt forgiveness has expansionary macroeconomic effects by enhancing aggregate demand formation.[8]

[7] Yet the plan to forgive student loans owned by the U.S. federal government has been met by several legal challenges. The main complaint is that the U.S. Department of Education is acting outside of its administrative authority by forgiving student loans it owns. As of currently (November 2022), the U.S. Department of Education has stopped taking applications for the student loan forgiveness plan after a federal judge ruled it is illegal.

[8] It is outside our scope here to fully explore the implications of cancellation or forgiveness of the type of debt considered in this paper. Using a macrodynamic model bearing some similarities to the present one and calibrated with U.S. data, Serra (2022) simulates the impact of a one-time one percentage-point cancellation in the ratio of student loan outstanding debt to gross domestic product. Owing to the rise in households' disposable income, capacity utilization increases in the short run. However, as households continue borrowing to finance expenses on education, the considered debt ratio converges to its original, pre-cancellation long-run equilibrium value.

5. Conclusion

The role of knowledge capital as a source of output growth in the long run has been increasingly explored in the literature. However, given that it is typically assumed that the long-run output growth is determined under conditions of full capacity utilization of the factors of production, among which knowledge capital is then included, it is unduly neglected the role of aggregate demand in output growth dynamics and the impact of autonomous investment in knowledge capital by working households in and of itself on aggregate demand formation. Meanwhile, demand-driven approaches to output growth dynamics have typically relegated closer attention to the accumulation of knowledge capital as narrowly supply-sided.

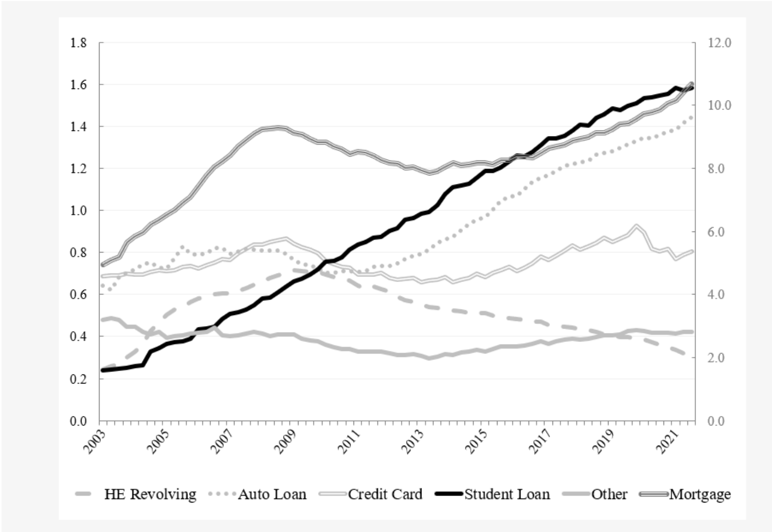
As the making of costly investments in human capital accumulation through education is indeed one of the main forms of productivity-enhancing knowledge accumulation, this paper explores several dynamic implications of debt-financed accumulation of knowledge capital, along with its resulting positive impact on labor productivity, within a demand-led macrodynamic model. Although our model does not focus specifically on debt-financed knowledge capital formation through student loans, the recent U.S. experience with student debt illustrates the significance for macroeconomic dynamics of the financing of knowledge capital accumulation by means of debt by working households. The model features knowledge capital formation as an additional source of aggregate demand alongside with expenditures in consumption and investment in physical capital. Given that the stock of knowledge capital remains uniformly distributed across workers, it follows that unemployed labor also means unutilized knowledge capital.

Although any increase in labor productivity brought about by knowledge capital accumulation is fully and automatically passed on to the real wage, so that the wage share in income remains constant, the employment rate is determined by aggregate demand. It follows that an exogenous increase in the wage share positively impacts on the short-run equilibrium rates of physical and knowledge capital utilization; in the long-run equilibrium, however, the same increase exerts a a positive impact on physical capital utilization and output growth but has an ambiguous impact on knowledge capital utilization. The long-run equilibrium features the equality between the stock of physical capital multiplied by its rate of utilization and the stock of knowledge capital multiplied by its rate of utilization. As a result, with a higher wage share yielding both a higher physical capital utilization and a lower ratio of physical capital to knowledge physical capital in the long-run equilibrium, the accompanying effect on the rate of knowledge capital utilization (which also measures the rate of employment) is ambiguous.

The long-run equilibrium configuration features several other interesting results, some of which are the following. First, the allocation of a higher proportion of wage income to debt repayment at the expense of consumption on the part of working households yields lower physical and knowledge capital utilization as well as output growth, and a higher debt ratio. Therefore, a more affordable debt service brings about a higher level of macroeconomic activity and a lower debt ratio. In fact, working households' debt ratio varies negatively with the wage share as well, while it is unaffected by a change in the exogenous value of the rate of interest. It follows that a better functional distribution of income in favor of wage-earners is effective in reducing their debt ratio, while a fall in the interest rate is not. Second, although there is no net debt repayment in any short run, so that the stock of debt always varies positively, in the long-run equilibrium (which is unique and stable) the rate of growth of output (which is the same as the rates of growth of physical and knowledge capital) is strictly greater than the interest rate. This result plays an important role in the stability of the unique long-run equilibrium configuration: the fact that the rate of growth of output is strictly greater than the interest rate means that the increase in the debt is dampened because output grows faster than the interest cost of the debt. Third, the desired investment in accumulation of physical and knowledge capital (the former done by capitalist households, the latter by working households) both feature an accelerator effect, with the former (latter) varying positively with the rate of physical (knowledge) capital utilization. In the long-run equilibrium, while a stronger accelerator effect acting in the desired investment in physical capital accumulation positively impacts on the rates of growth of both physical and knowledge capital and therefore output, the accompanying impact on the rate of physical capital utilization is ambiguous. Similarly, in the long-run equilibrium, although a stronger accelerator effect operating in the desired investment in knowledge capital formation positively impacts on the rates of growth of physical and knowledge capital and hence output, the accompanying impact on the rate of knowledge capital utilization (or rate of employment) is ambiguous. Yet a stronger accelerator effect in the desired investment in accumulation of physical (knowledge) capital positively impacts on the rate of knowledge (physical) capital utilization in the long-run equilibrium. Interestingly, therefore, a stronger accelerator effect in the desired investment in one of the two types of capital increases the rate of utilization of the other type but reduces its own rate of utilization in the long-run equilibrium.

FIGURES

Figure 1: Household debt balance in the U.S.: total and composition (in trillions of dollars)*

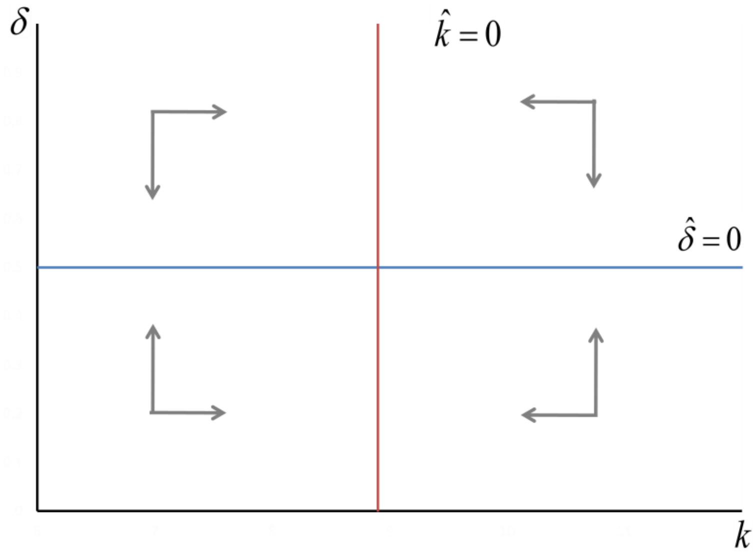


Source: FRBNY Consumer Credit Panel / Equifax.

*HE Revolving: Home Equity Revolving.

Note: Values for Mortgage are represented in the right vertical axis, while values for the other debt types are represented in the left vertical axis.

Figure 2: Stable long-run equilibrium



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Appendix A: *Wage income for debt repayment as a proportion of the physical capital stock*

We can show that the proportion of the wage income that working households allocate to debt repayment, $(1-\phi)VL$, is greater than, equal to or lower than the interest payment due, iD, in a given short run. Let us scale the expression $(1-\phi)VL - iD$ by the physical capital stock, which yields $\Phi = (1-\phi)\sigma u - i\delta$. Using (17), we get:

$$\Phi = i\delta\left[\frac{\sigma(1-\phi)(1-s)}{\Omega} - 1\right]. \qquad \text{(A-1)}$$

Therefore, for a given strictly positive value of the state variable δ, which will be the case in the long-run equilibrium, the wage share $\bar{\sigma}$ which satisfies $\Phi(\bar{\sigma}) = 0$ is given by:

$$\bar{\sigma} = \frac{s-\beta}{s\phi+\gamma}. \qquad \text{(A-2)}$$

Notice that $\Omega > 0 \Rightarrow s(1-\sigma)-(\phi+\gamma-1)\sigma-\beta > 0 \Rightarrow s-\beta > 0$ (recalling our assumption that $\phi+\gamma > 1$), so that $\bar{\sigma} > 0$. We assume that $\bar{\sigma} < 1$, which requires that $s < (\beta+\gamma)/(1-\phi)$. Also, we have:

$$\frac{\partial\bar{\sigma}}{\partial s} = \frac{\beta+\gamma}{(s\phi+\gamma)^2} > 0,$$

$$\frac{\partial\bar{\sigma}}{\partial\beta} = -\frac{1}{s\phi+\gamma} < 0,$$

$$\frac{\partial\bar{\sigma}}{\partial\phi} = -\frac{s(s-\beta)}{(s\phi+\gamma)^2} < 0,$$

and:

$$\frac{\partial\bar{\sigma}}{\partial\gamma} = -\frac{s-\beta}{(s\phi+\gamma)^2} < 0.$$

Appendix B: *Positivity of the unique long-run equilibrium ratio of debt to physical capital*

We can show that the strict positivity of the unique long-run equilibrium value of the ratio of working households' debt to physical capital δ^* in (28) is an mplication of our assumptions that $s<1$ (which ensures that the denominator of δ^* is strictly positive) and $\phi+\gamma>1$ (which ensures that the numerator of δ^* is strictly greater than $\Omega>0$). We can prove that the numerator of δ^* (and hence the whole expression for δ^*) is strictly positive as follows:

$$\begin{aligned}
&\phi+\gamma>1 \Rightarrow (1-s)(\phi+\gamma)>(1-s) \Rightarrow -s(\phi+\gamma)>(1-s)-(\phi+\gamma) \Rightarrow \\
&\Rightarrow -s(\phi+\gamma)\sigma>(1-s)\sigma-(\phi+\gamma)\sigma \Rightarrow \\
&\Rightarrow s-s(\phi+\gamma)\sigma-\beta>s+(1-s)\sigma-(\phi+\gamma)\sigma-\beta \Rightarrow \\
&\Rightarrow s\left[1-(\phi+\gamma)\sigma\right]-\beta>\Omega .
\end{aligned} \tag{B-1}$$

Let us compute how the long-run equilibrium value of the ratio of working households' debt to physical capital in (28) varies with each of the parameters of which it is a function. We have:

$$\frac{\partial\delta^*}{\partial\phi}=\frac{\partial\delta^*}{\partial\gamma}=-\frac{s\sigma}{\beta(1-s)}<0, \tag{B-2}$$

$$\frac{\partial\delta^*}{\partial\sigma}=-\frac{s(\phi+\gamma)}{\beta(1-s)}<0, \tag{B-3}$$

and:

$$\frac{\partial\delta^*}{\partial s}=\frac{1-(\phi+\gamma)\sigma-\beta}{\beta(1-s)^2}>0 . \tag{B-4}$$

Recalling that $\Omega>0$, we can prove that the expression in the numerator in (B-4) is strictly positive as follows:

$$\begin{aligned}
&1-(\phi+\gamma)\sigma-\beta>\Omega \Rightarrow 1-(\phi+\gamma)\sigma-\beta>s(1-\sigma)+\sigma-(\phi+\gamma)\sigma-\beta \Rightarrow \\
&\Rightarrow (1-\sigma)>s(1-\sigma) \Rightarrow 1>s,
\end{aligned} \tag{B-5}$$

which is our assumption about the capitalists' saving rate.

$$\frac{\partial \delta^*}{\partial \beta} = -\frac{s\left[1-(\phi+\gamma)\sigma\right]}{\beta^2(1-s)} < 0\,. \tag{B-6}$$

As the expression in the numerator in (B-4) is strictly positive, the expression in brackets in the numerator in (B-6) is strictly positive as well.

$$\frac{\partial \delta^*}{\partial i} = 0\,. \tag{B-7}$$

This interest rate neutrality result arises because the long-run equilibrium values of the growth rate of the debt, $\hat{D}^*$, and the growth rate of physical capital, $\hat{K}^*$, respond to a change in the interest rate with a change in the same direction and of the same magnitude. In effect, using (7), (17) and (28) in (26), we get:

$$\frac{\partial \hat{D}^*}{\partial i} = \frac{(\phi+\gamma-1)(1-s)\sigma+\Omega}{\Omega} > 0 \tag{B-8}$$

and:

$$\frac{\partial g^*}{\partial i} = \frac{\partial g_K^*}{\partial i} = \frac{s[1-(\phi+\gamma)\sigma]-\beta}{\Omega} > 0\,, \tag{B-9}$$

where the strictly positive sign in (B-8) and (B-9) is per (B-1). It can be demonstrated that the magnitude of both such effects is the same:

$$\begin{aligned} & s\left[1-(\phi+\gamma)\sigma\right]-\beta-\Omega = s-s(\phi+\gamma)\sigma-\beta-s-(1-s)\sigma+(\phi+\gamma)\sigma+\beta = \\ & = -s(\phi+\gamma)\sigma-(1-s)\sigma+(\phi+\gamma)\sigma = (1-s)(\phi+\gamma)\sigma-(1-s)\sigma = \\ & = (\phi+\gamma-1)(1-s)\sigma. \end{aligned} \tag{B-10}$$

Appendix C: *Comparative statics of the macroeconomic variables in the long-run equilibrium*

Substituting (28) in (17), the impact of a change in the capitalists' saving rate on the long-run equilibrium value of the rate of physical capital utilization is given by:

$$\frac{\partial u^*}{\partial s} = -\frac{\left[1-(\phi+\gamma)\sigma-\beta\right](\phi+\gamma-1)i\sigma}{\beta\Omega^2} < 0, \qquad \text{(C-1)}$$

the sign of which is strictly negative given our assumptions that $s<1$, $\phi+\gamma>1$ and $\Omega>0$. Recall that we have proved above that the expression in the numerator in (B-4), which is the same as the expression in brackets in the numerator in (C-1), is strictly positive, so that the whole expression in (C-1) is strictly negative. The paradox of thrift also applies to the long-run equilibrium values of the rates of output growth and employment, in the latter case recalling that k^* does not depend on the capitalists' saving rate.

The impact of a change in the consumption coefficient ϕ in (13) on the long-run equilibrium value of the rate of physical capital utilization is given by:

$$\frac{\partial u^*}{\partial \phi} = \frac{\partial u^*}{\partial \gamma} = \frac{\left[s(1-\sigma)-\beta\right](1-s)i\sigma}{\beta\Omega^2} > 0, \qquad \text{(C-2)}$$

the sign of which is strictly positive given our assumptions that $s<1$, $\Omega>0$ and $\phi+\gamma>1$. In fact, $\Omega>0 \Rightarrow s(1-\sigma)-(\phi+\gamma-1)\sigma-\beta>0 \Rightarrow s(1-\sigma)-\beta>0$. A lower (higher) consumption (debt repayment) coefficient out of wage income yields lower long-run equilibrium values of the physical capital utilization, output growth and employment rate (in the latter case recalling that k^* does not depend on ϕ).

The response of the long-run equilibrium physical capital utilization to a change in the wage share is given by:

$$\frac{\partial u^*}{\partial \sigma} = \frac{(1-s)(\phi+\gamma-1)(s-\beta)i}{\beta\Omega^2} > 0, \qquad \text{(C-3)}$$

the sign of which is strictly positive given our assumptions that $s<1$, $\Omega>0$ and $\phi+\gamma>1$. In fact, $\Omega>0 \Rightarrow s(1-\sigma)-(\phi+\gamma-1)\sigma-\beta>0 \Rightarrow s-\beta>0$. Given (7), a higher wage share yields a higher long-run equilibrium value of the rate of output growth as well.

The response of the long-run equilibrium physical capital utilization to a change in the other parameter (in addition to the wage share) denoting the strength of the accelerator effect in the desired rate of growth of knowledge capital, γ, is also given by (C-2). The long-run equilibrium values of the physical capital utilization and output growth also vary positively with such other parameter determining the sensitivity of workers' desired rate of growth of knowledge capital to the rate of employment, while (per (B-2)) the household debt ratio varies negatively with it in the long-run equilibrium. Nonetheless, an increase in the sensitivity of workers' desired rate of growth of knowledge capital to the rate of employment also raises the ratio of knowledge to physical capital in the long-run equilibrium, which is given by k^{*-1}. As a result, the long-run equilibrium rate of employment does not necessarily vary positively with any of the parameters (σ and γ) determining the sensitivity of workers' desired rate of growth of knowledge capital to the rate of employment, which also measures the strength of the accelerator effect in workers' desired rate of growth of knowledge capital. Substituting (27) and (28) in (18), we get:

$$\frac{\partial e^*}{\partial \sigma}=\frac{i}{\gamma\sigma^2\Omega^2}\left\{(s-\beta)\left[(1-s)(\phi+\gamma-1)\sigma-\Omega\right]+s(\phi+\gamma)\sigma\Omega\right\} \qquad \text{(C-4)}$$

and:

$$\frac{\partial e^*}{\partial \gamma}=\frac{i}{\sigma\gamma^2\Omega^2}\left\{\beta\left[\Omega-\gamma\sigma(1-s)\right]+s\left[\gamma\sigma(1-s)(1-\sigma)\right]+s\Omega\left[(\phi+\gamma)\sigma-1\right]\right\}. \qquad \text{(C-5)}$$

Recall that the short-run equilibrium rates of physical capital utilization and employment in (17) and (18), respectively, vary positively with the parameter measuring the strength of the accelerator effect in the desired rate of accumulation of physical capital, which is β in (7). In the short run, the debt ratio, δ, and the physical capital to knowledge capital ratio, k, are given, but in the long-run equilibrium the former (latter) varies negatively (positively) with β. Thus, the response of the long-run equilibrium rates of utilization of physical and knowledge capital and output growth to a change in β is mediated by effects in different directions, but which nonetheless does not yield an ambiguous result for most of such macroeconomic variables. Recall from (7), using (17) and (28), that the output growth rate in the long-run equilibrium is given by:

$$g^* = \beta u^* = \frac{\beta(1-s)i\delta^*}{\Omega}. \quad \text{(C-6)}$$

Therefore, it follows that:

$$g^*_\beta = \frac{\partial g^*}{\partial \beta} = \frac{(1-s)i}{\Omega^2}\left[(\beta+\Omega)\delta^* + \beta\Omega\delta^*_\beta\right], \quad \text{(C-7)}$$

where $\delta^*_\beta = \partial\delta^* / \partial\beta < 0$. We can use either (28) or (B-6) to obtain that $\beta\delta^*_\beta = -\left(\delta^* + \frac{1}{1-s}\right)$, the substitution of which in (C-7) yields:

$$g^*_\beta = \frac{(1-s)i}{\Omega^2}\left[\beta\delta^* - \frac{\Omega}{1-s}\right] = \frac{1}{\Omega}\left[\frac{\beta(1-s)i\delta^*}{\Omega} - i\right] = \frac{g^* - i}{\Omega} > 0. \quad \text{(C-8)}$$

The impact of a change in the parameter measuring the strength of the accelerator effect in the desired rate of accumulation of physical capital on the long-run equilibrium value of the output growth rate (which is also the long-run equilibrium value of the growth rate of physical capital in this economy producing a single good) is apparently ambiguous. However, it can be shown that our assumption that $\phi + \gamma > 1$ implies that the long-run equilibrium value of the output growth rate is strictly greater than the interest rate, so that the sign of the expression in (C-7) is strictly positive:

$$\begin{aligned}&\phi+\gamma>1\Rightarrow(1-s)(\phi+\gamma)>(1-s)\Rightarrow -s(\phi+\gamma)\sigma>(1-s)\sigma-(\phi+\gamma)\sigma\Rightarrow\\&\Rightarrow s-s(\phi+\gamma)\sigma-\beta>s+(1-s)\sigma-(\phi+\gamma)\sigma-\beta\Rightarrow\\&\Rightarrow s\left[1-(\phi+\gamma)\sigma\right]-\beta=\beta(1-s)\delta^{*}>\Omega\Rightarrow\beta(1-s)i\delta^{*}>i\Omega\Rightarrow g^{*}>i.\end{aligned}\tag{C-9}$$

Using (B-6), the response of the long-run equilibrium output growth rate to a change in β can be alternatively expressed as:

$$g_{\beta}^{*}=\beta u_{\beta}^{*}+u^{*},\tag{C-10}$$

where $u_{\beta}^{*}=\partial u^{*}/\partial\beta$, which, rearranging (C-10) and using (C-8), can be expressed as:

$$u_{\beta}^{*}=\frac{g_{\beta}^{*}-u^{*}}{\beta}=\frac{1}{\beta}\left(\frac{\beta u^{*}-i}{\Omega}-u^{*}\right)=-\frac{1}{\beta\Omega}\left[i+(\Omega-\beta)u^{*}\right],\tag{C-11}$$

which is ambiguous in sign. Despite the ambiguity in the impact of a change in the parameter which indicates the strength of the accelerator effect in the desired rate of physical capital accumulation on the long-run equilibrium physical capital utilization, the impact of the same change on the long-run equilibrium growth rate of physical capital (which is also the long-run equilibrium growth rate of output) is strictly positive. In fact, if the impact expressed in (C-11) is strictly negative, the absolute value of the respective elasticity measure is strictly lower than one. This can be readily seen by re-writing the strictly positive expression in (C-10) in terms of this elasticity measure as follows:

$$g_{\beta}^{*}=(\varepsilon+1)u^{*},\tag{C-12}$$

where $\varepsilon=(\beta/u^{*})u_{\beta}^{*}$ is the β elasticity of u^{*}. Using (17) and (C-11), we get:

$$\varepsilon=-\frac{1}{(1-s)\delta}-\frac{\Omega-\beta}{\Omega},\tag{C-13}$$

which is strictly negative if $\Omega > \beta$, which in turn implies that u_{β}^{*} in (C-11) is strictly negative as well. In any case, it can be shown, using (28), that our assumptions that $s < 1$ and $\phi + \gamma > 1$ ensures that the absolute value of ε is strictly lower than one:

$$
\begin{aligned}
&|\varepsilon| < 1 \Rightarrow \left| -\frac{1}{(1-s)\delta} - \frac{\Omega - \beta}{\Omega} \right| < 1 \Rightarrow \Omega + (1-s)(\Omega - \beta)\delta < (1-s)\Omega\delta \Rightarrow \\
&\Rightarrow \Omega - (1-s)\beta\delta < 0 \Rightarrow \Omega - s[1-(\phi+\gamma)\sigma] + \beta < 0 \Rightarrow \\
&\Rightarrow -s\sigma + \sigma - (\phi+\gamma)\sigma + s(\phi+\gamma)\sigma < 0 \Rightarrow (1-s)\sigma - (1-s)(\phi+\gamma)\sigma < 0 \\
&\Rightarrow (1-s)(1-\phi-\gamma)\sigma < 0.
\end{aligned} \tag{C-14}
$$

Finally, despite the ambiguity in the impact of a change in the parameter measuring the strength of the accelerator effect in the desired rate of physical capital accumulation on the long-run equilibrium physical capital utilization, the impact of such a change on the long-run equilibrium rate of employment (which also measures the long-run equilibrium value of knowledge capital utilization) is strictly positive as well. As the long-run equilibrium employment rate is given by $e^{*} = u^{*}k^{*}$ (per (B-6)), and the long-run equilibrium physical capital to knowledge capital ratio k^{*} varies positively with β (see (27)), so that the availability of knowledge capital relatively to physical capital varies negatively with β, an increase in the latter ultimately raises the long-run equilibrium rate of knowledge capital utilization. Substituting (27) and (28) in (18), we can then compute:

$$
\frac{\partial e^{*}}{\partial \beta} = \frac{i\left[(1-s)(\phi+\gamma-1)\right]}{\gamma\Omega^{2}} > 0, \tag{C-15}
$$

the sign of which is also implied by our assumptions that $s < 1$ and $\phi + \gamma > 1$.