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Abstract

Robert E. Lucas Jr. is considered the “architect” of modern macroeconomics. His equilibrium approach to the business cycles has provoked a major change in the understanding of macroeconomic phenomena since the late 1960s. In this article we attempt to describe historically how he put together the main elements that formed the body of his theoretical framework, namely, the optimizing representative agent, the contingent-claim approach to general equilibrium analysis and the modeling of the rational expectation hypothesis. Lucas’ *Expectations and the Neutrality of Money*, published in 1972, is the first article containing all elements aforementioned. To reach such a result, he collaborated with Leonard Rapping in 1969 (their first article) and later developed a joint work with Edward Prescott in 1971. Furthermore, we also argue that the way Robert Lucas saw business cycles can be considered an inevitable progress in macroeconomics.

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**JEL Codes:** B2, B22
Lucas’ Equilibrium Account of the Business Cycle: Optimizing Behavior, General Equilibrium, and Modeling Rational Expectations¹

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ABSTRACT

Robert E. Lucas Jr. is considered the “architect” of modern macroeconomics. His equilibrium approach to the business cycles has provoked a major change in the understanding of macroeconomic phenomena since the late 1960s. In this article we attempt to describe historically how he put together the main elements that formed the body of his theoretical framework, namely, the optimizing representative agent, the contingent-claim approach to general equilibrium analysis and the modeling of the rational expectation hypothesis. Lucas’ Expectations and the Neutrality of Money, published in 1972, is the first article containing all elements aforementioned. To reach such a result, he collaborated with Leonard Rapping in 1969 (their first article) and later developed a joint work with Edward Prescott in 1971. Furthermore, we also argue that the way Robert Lucas saw business cycles can be considered an inevitable progress in macroeconomics.

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¹ Preliminary and incomplete version. Comments are welcome.
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1. Introduction

Robert E. Lucas Jr. is widely regarded by the majority of the profession as the most influential macroeconomist since the late 1960s, having his works reshaped the way business cycles are comprehended. The main line of argument pursued by Lucas was that the “recurrent character of business cycles” should never have been diverted from the original interpretation of the interwar economists, particularly, Friedrich von Hayek’s view by which the incorporation of cyclical phenomena into the system of general economic equilibrium, although in apparent contradiction, is the crucial problem of trade theory and therefore a significant challenge for economists.

Indeed, Lucas not only took up that challenge but also, as a by-product of his research agenda, pushed further economists’ understanding of the correct way to evaluate economic policies. As will be argued in this essay, Lucas succeeded in solving the problem proposed by Hayek through a combination of old and new neoclassical assumptions, such as the optimizing behavior of a representative agent and the contingent-claim hypothesis of the Arrow-Debreu-McKenzie general equilibrium theory, respectively, along with a novel way of introducing rational expectations into the framework.

In other words, for Lucas (1977, 1980), besides the cyclicality of the phenomena at hands, which provides invaluable insights to economists, the decisive advance toward its understanding was the development of new theoretical methods, for in the author’s view: “There is no reason to view our ability to think as being any less limited by available technology than our ability to act” (Lucas, 1980, p. 286). This essay aims at reconstructing the historical context by which the economic and mathematical ideas came to be amalgamated by Robert Lucas since the end of the 1960s and which turned out to be the New Classical research agenda.

Apart from this introduction, the article has four more sections. Section two explores Lucas’ (and Leonard Rapping’s) attempt at formulating a macroeconomic

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1 In the article “Understanding Business Cycles”, published in 1977, Robert Lucas points to F. A. Hayek (1933) as the main “intellectual ancestor” behind his rationale for pursuing the understanding of business cycles as equilibrium phenomena. In this paper we simply take such a point of view for granted and build the historical context by which we understand Lucas succeeded. However, we do recognize the controversy it generated. Kevin Hoover (1988) dedicated an entire chapter of his book to analyzing whether the New Classical research program was indeed an “Austrian Revival.” Also, in a good survey of the issue, Jorge Soromenho (1998) rejects the idea. Interestingly, in an interview, Lucas (1998) himself admits that Hoover convinced him that he had misread Hayek.
model where representative agents (firm and consumer) behave optimally the whole time, that is, maximize their profit and utility functions continually in the context of a general equilibrium framework. Section three describes how in later articles he used a more sophisticated general equilibrium model, that with the contingent-claim hypothesis, to better his approach after Edmund Phelps (1970) made him realize his model was incomplete. Section four discusses how his 1971 paper with Edward Prescott provided him with the ultimate tools to finally build a macroeconomic theory erected under a general equilibrium framework, materialized in his well known “Expectation and the Neutrality of Money” published in 1972. In this section we also give a brief historical overview of how the dynamic programming developed by Richard Bellman enabled the writings of the two papers just mentioned. Section five presents some final thoughts.

2. Lucas and Rapping’s Attempt at Modeling an Optimizing Representative Agent within a General Equilibrium Framework

Robert Lucas and Leonard Rapping published two important and interrelated papers in 1969. The first article “Real Wage, Employment and Inflation” (1969a) came out in the Journal of Political Economy, while the second “Price Expectations and the Phillips Curve” (1969b) was published in the American Economic Review. The two papers appeared while both authors were still associate professor of economics at the Carnegie-Mellon University4. In those two articles, Lucas and his co-author inaugurated a new tradition in macroeconomics, a theoretical development that later became known as the new classical school of economic thought. According to Hoover (1988) the seed of such rationale came from Milton Friedman’s now famous presidential address at the American Economic Association in 1967 and later published as an article in the American Economic Review in 1968 entitled “The role of monetary policy.”

In that paper, Milton Friedman reinterpreted the Phillips curve, a recently incorporated tool to the Keynesian theoretical framework5, using neoclassical principles and reached a different conclusion. That is, the short run Phillips

4 Robert Lucas moved to the University of Chicago in 1975 and remains there up to this day. Leonard Rapping moved to the University of Massachusetts in 1982 and died in 1991 at the age of 57.
5 Hoover (1988) argues that the Phillips curve was adopted by the Keynesians in the 1960s as a tool to explain inflation. The reason is that before WWII, in a historical context of high unemployment, inflation was not a problem resulting in a lack of adequate theorizing on the subject.
curve is feasible as long as workers do not understand that a rise in nominal price – caused by an increase in aggregate demand through an expansionary monetary policy, for example – is in fact lowering their real wage: although their money wage is increasing they cannot keep up with the price level. As a result, they supply more labor and unemployment falls (firms hired more workers). But Friedman argued that workers’ mistakes do not happen indefinitely and sooner or later they adjust their price expectations or the future course of inflation. The resulting demand for higher money wages from workers will make firms to cut back on productions, dismiss labor and thus bring the economy back to a natural rate of unemployment (the unemployment rate before government intervention).

The natural rate therefore is the equilibrium rate of unemployment to which the stable private economy tends to return once disruptive influences, such as attempts of governments to manipulate aggregate demand, are removed. Friedman believes that the economy is usually reasonably close to the natural rate and, when displaced from it, tends to return to it. (Hoover 1988: 25)

Some interesting events took place after this episode. First, following Friedman’s criticism of the Phillips curve as proposed by Samuelson and Solow (1960), the new version was restated as “the expectation-augmented” Phillips curve in the literature. Such a version became therefore associated to the possible existence of a vertical graph coupled with the notion of a natural rate of unemployment. A significant part of the Keynesians versus Monetarists debate in the 1970s centered on the existence of this long run or vertical version of the theory (Snowdon and Vane, 2005, p. 178-9). Second, and directly related to our work, even the Keynesians who adopted the expectations-augmented version of the Phillips curve downplayed the message of equilibrium nature of unemployment implicit in Friedman’s analysis. In other words:

Unemployment was a disequilibrium phenomenon, but the source of the disequilibrium was the failure of individual workers (and perhaps) firms to form accurate expectations of the future course of inflation. Given their expectations, their choices were optimal. (Hoover 1988: 26)

Lucas and Rapping (1969a) did not address the issue of the Phillips Curve explicitly, for their focus was on the correlation between price levels and employment. Nevertheless, it showed that in the short run equilibrium was
attainable in the labor market through workers’ utility and firms’ profit maximization processes. In the long run, given the authors’ modeling of the expectations (adaptive) no change in the labor supply occurred at all and hence (even considering that it was not their focus) it could be interpreted as if the Phillips curve disappeared. Such a result provided a more rigorous, model-based, way to corroborate with Milton Friedman’s (1968) and Edmund Phelps’ (1968) ideas.

What can be observed from the intellectual context above is that Lucas and Rapping (1969a) did not only try to interpret the Phillips curve as consistent with equilibrium in the labor market, but also introduced the artifice of an optimizing representative agent into macroeconomic modeling. In other words, the 1969a paper consisted as a first incursion into a new development in theoretical macroeconomics since it raised the property of market clearing in the labor market as well as pushed for the modeling of a representative agent’s optimization problem. The focus of this section is therefore on what consisted of Lucas’ efforts to carry on this research agenda. Lucas did this by also launching harsh criticism of the Keynesian macroeconomics, according to which they (the Keynesians) tried to provide microfounded models of the macroeconomy in the 1960s, but “(...) fell short of incorporating the discipline of the optimization problem into the general equilibrium framework” (Hoover, 2012, p. 20).

In 1981, Lucas published a book entitled *Studies in Business-Cycle Theory*, a compilation of his most important papers on the subject written up until that year. In the introductory chapter of the volume it is possible to get a flash of the rationale behind his engagement with microfounded models in general and with optimizing representative agent in particular.

My first research directly related to business cycle was a joint project undertaken in collaboration with Leonard Rapping, “Real Wages, Employment, and Inflation,” on the determination of aggregated employment and wages in the US economy. Our objective at the time, as

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6 Edmund Phelps (1967, 1968) should also be credited with having reached the same conclusion that did Milton Friedman.

7 The microfoundation literature is an extremely rich and contentious one and is certainly beyond the scope of this paper. The reason is that our approach is restricted to the discussion of models consisted of representative agents who solve optimization problems and not their methodological or historical merits for macroeconomic modeling. Many important books address this issue with the necessary depth and width: Weintraub (1979), Kirman (1992), Janssen (1993), King (2012) and Duarte and Lima (2012). Many important papers also contribute to the subject: Weintraub (1977), Backhouse and Boianovsky (2006), Duarte and Lima (2012), Duarte (2012), Hoover (2012), just to name a few examples.
is clear enough from the introduction to our paper and such defensive efforts as note 3, was not to challenge the current, Keynesian orthodoxy but rather to contribute to it by constructing a “*micro-economic foundation*” for the wage-price sector of macroeconometric models. Our strategy in attempting this was to try to *rationalize the employment decisions of individuals households and firms* in a manner modeled after Friedman and Modigliani’s work on the household consumption decision, Eisner and Jorgenson’s work on the investment decision of business firms, or Friedman and Meltzer’s work on the demand for money. (Lucas 1981: 2-3, emphasis added)

Notwithstanding Lucas recognized the microfoundation project taken up by earlier Keynesians, for him it was of limited use. A leading scholar in New Classical macroeconomics explains that for Lucas:

The microeconomics of the various functions mainly served to suggest a list of regressors to “explain” their target variables. The regressors themselves were, in effect, merely rules of thumb – decisions rules for particular environments. Lucas (1977: 220-21) did not deny that macroeconometric models constructed in this manner could well mimic the behavior of the actual economy, but appealing to the main theme of his famous article “Econometric Policy Evaluation: A Critique” (1976), he argued that the regressions would not isolate the invariants in the economy and that conditional forecasting (policy analysis) requires such invariants. (Kevin Hoover 2012: 20)

It is important to notice that even though Lucas’ own work with Rapping could be framed into that “old-style microfoundation”, since it was doing nothing more than providing the microeconomic bases for a Keynesian function⁸, Hoover (2012, p. 46) calls attention to the fact that their first 1969 article: “(...) has strong claims to be the first paper in the New Classical traditional and the fount of the representative agent program”. The very first difference between Lucas and Rapping’s (microfoundation) research program and those before them was the explicit use of the artifice of a representative agent. The second contribution of that seminal paper was the attempt at connecting explicitly such microeconomic-optimizing-behavior of economic agents to fluctuations in wages and prices and, in such a way, by understanding the responses of economic agents to cyclical movements of economic variables as optimal decisions, they advanced toward the formulation of a general equilibrium account of the business cycle. By doing this

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⁸ Lucas and Rapping (1969a) derive an aggregate labor supply function through household decisions involving the choice between work and leisure.
Lucas was moving a step closer to the challenge posed by Hayek (1933) and raised in his 1977 paper.

We were aware, of course, that working toward a competitive-equilibrium account of the cyclical behavior of employment would be viewed in some circles as quixotic or even subversive, but we supposed that the basis for this widespread view was no deeper than the belief that such an account was not possible to devise. Since each of us had had some classroom success in discussing the behavior of employment over the cycle in price-theoretical terms, the task did not seem at all forbidding, and we proceeded with it. (Lucas 1981: 2-3)

In other words, it can be claimed here that Lucas and Rapping’s innovative approach does not only stem from their contribution to the rise in importance of microeconomics to the study of macroeconomic phenomena, by way of a representative agent, sometimes considered as a standard narrative in the microfoundation literature, but as well as from their effort to show how it could be integrated into a general equilibrium framework. It can also be claimed that no theoretical innovation precedes another. It is interesting to observe then that the microeconomic-optimizing-behavior and the general equilibrium approach to macroeconomics seem to be completely internalized in today’s macroeconomic theory. The attachment of the first to the second (or otherwise) in a modeling process became almost natural to the extent that Backhouse and Boianovský (2006), in writing about modern macroeconomics, affirm that:

Microeconomic theories of individual optimization are central to the main results that differentiate contemporary macroeconomics from the already-forgotten theories of the decades immediately following the Second World War. (Backhouse and Boianovský 2006: 1)

The statement by Backhouse and Boianovský (2006) seems very up to date. In fact, in the 1980s, a little more than a decade after Lucas and Rapping (1969a) published their paper, the macroeconomics landscape seemed to have changed already in favor of the now established new classical macroeconomics school. Thomas Sargent, also a leading scholar in the new movement, wrote in his 1982 paper “Beyond Demand and Supply Curves in Macroeconomics”, published in the American Economic Review, that in this new approach:

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9 Duarte and Lima (2012) analyze this point clearly. Complementing Duarte and Lima’s (2012) article is Hoover’s (2012) paper on various microfoundational programs that existed prior to Lucas and Rapping’s incursion into the field.
(...) the content of *optimizing dynamic economic theory* was to deliver cross-equations restrictions across the distributed lags in decision rules, on the one hand, and the equations for the motion of the variables that appear in *agent*'s objective functions and which they care about predicting, on the other. (Thomas Sargent 1982: 382, emphasis added)

At the same time, the cited author continued, the investigative method “(...) drives the analyst toward explicitly formulating dynamic general equilibrium models at the level of objective functions, constraints sets, and market-clearing conditions of their counterparts” (Sargent, 1982, p. 382). In this last passage it is clear that it contains contributions from Edward Prescott to the new classical literature in terms of rational expectation modeling, an issue I will tackle in section four. Even without explicitly mentioning that exists a general equilibrium framework behind microeconomic theories of individual optimization, those familiar with such “modern macroeconomics” recognize it immediately. In fact, the opposite could also be shown to be true: today’s core of macroeconomics, called dynamic stochastic general equilibrium (DSGE) model, has in it the term “general equilibrium” and not a representative agent, but likewise its microeconomic traits are pervasive.


“Real Wage, Employment and Inflation” (1969a) became a watershed in the macroeconomics literature by incorporating a microfoundation program based upon a representative agent as well as by Lucas and Rapping’s strive to make widespread use of the concept of general equilibrium theory to the understanding of the business cycle. In this section, the objective is to get a little deeper into the historical developments related to this latter research. I will argue in this section that the term “general equilibrium” initially used by the authors had to be better qualified before they could effectively make use of it. As a matter of fact, Lucas (1981: 6) recalls that the best thing that happened to his paper after it was published was that Edmund Phelps came across the article and decided to include it in a book he was editing called “Microeconomic Foundation of
Employment and Inflation Theory”. The book came out in 1970. In a conference organized to celebrate the publication Phelps, according to Lucas, raised the issue of general equilibrium theory by which no paper had satisfactorily addressed, including the Lucas’ (and Rapping’s) 1969 paper. In the volume:

Phelps’ introductory essay dealt with this general-equilibrium issue in an informal but concrete way. Much of our discussion at the conference involved questions that seemed to stand in the way of casting this argument in modern mathematical form. (Lucas 1981: 7)

Then, Lucas explains that:

“Expectations and the Neutrality of Money” was an attempt to do this. This paper’s ties to the Phelps essay turned to be stronger than I had seen when I began work on it. The initial idea simply was to situate some Lucas-Rapping households in a monetary economy, subject the system to stochastic shocks to the money supply, and see what would happen. Samuelson’s intergenerational monetary economy offered a convenient analytical setting because it both fit the two-period consumer maximization problem Rapping and I had studied and the function of “money” in it was so clearly defined. The idea of defining an equilibrium as a point in a space of functions of a few “state variables” was one that Prescott and I had utilized in “Investment under Uncertainty.” This analytical device had forced Prescott and me to be precise as to the meaning of terms like information and expectations and led us to formulate and utilize Muth’s rational expectations hypothesis in exactly the way I then used it in “Expectations and the Neutrality of Money.” In short, the needed ingredients for a general-equilibrium formulation seemed to be readily at hand.” (Lucas 1981: 7)

In hindsight two messages seem clear, one from each of the passage above. First, despite Lucas and Rapping’s incursions at comprehending business cycles in a general equilibrium framework at the end of the 1960s the task had been an incomplete one. Second, prior to the writing of his 1972 paper, where he fully applied a mathematical tool to the utilization of Muth’s rational expectation (discussed in the penultimate section), Lucas pushed for the use of the Arrow-Debreu intertemporal (or with contingent-claim hypothesis) general equilibrium in macroeconomics. In other words, between his first incursion into the “discipline of optimization problems”, as summarized in his 1969a paper, and the point where all needed ingredients to formulate a general equilibrium framework
was available, understood as his 1972 article, Lucas tried to work out the contingent-claim hypothesis in macroeconomics. It paid off once:

Although Lucas frequently makes reference to general-equilibrium and to the Arrow-Debreu model, he applies very little of the technical work in general equilibrium theory to his macroeconomic models. His references tend to be casual, in much the same way as Friedman (1968: 8) famously defines the natural rate of unemployment by reference to the Walrasian general-equilibrium model, while never actually deriving a natural rate in such a model. Lucas’s “Expectations and the Neutrality of Money” (1972a) is an exception. It is a general-equilibrium model in the spirit of Arrow and Debreu. (Hoover 2012: 48)

In fact, in Lucas’ view, and cited elsewhere (introduction), it is natural the fact that available tools in microeconomic theory and the Arrow-Debreu framework (with contingent-claim hypothesis) in particular be put to good use in macro theory. Lucas:

(...) was willing to excuse the theoretical choices of Keynesian economists as the product of the exigencies of the great depression and the absence of appropriate tools. But economists after the development of the Arrow-Debreu contingent-claims framework can no longer be excused. New classical microfoundations begins with the optimization problems of individual agents. These are incorporated into a dynamic general-equilibrium model based in the contingent-claims framework. (Hoover 2012: 20)

What is the contingent-claim hypothesis anyway? In the Arrow-Debreu static general equilibrium paradigm agents’ preferences are defined over goods consumed in the present. In an intertemporal or dynamic setting, agents’ preferences are defined over goods deliverable in all future time periods and for each possible state of nature (Donaldson and Selden, 1981). Such future markets are defined in the literature as contingent markets and its validity the fulfillment of the “contingent-claim hypothesis”. Robert Lucas argues that there exist two ways to interpret a contingent claim equilibrium. The first one states that:

(The contingent claim equilibrium) is as a description of an economy in which all state-contingent prices are determined in advance, in the clearing of a single grand future market. On this interpretation, individual traders may assess the probabilities of the occurrences of future states of nature, but with prices determined in advance, the issue of expectation does not arise. (Lucas 1980: 285).
Alternatively, still according to the author, the second interpretation states that:

(The contingent claim equilibrium) is as being determined via a sequence of “spot” markets, in which current prices are set given certain expectations about future prices. On this second interpretation, one needs a principle to reconcile the price distributions implied by the market equilibrium with the distributions used by agents to form their own views of the future. (Lucas 1980: 285).

In Lucas’s view it was John Muth (1961) who formulated the principle for such a reconciliation. The afterward developed notion by which both distributions could not differ in systematic way was called “rational expectation”. As will be explored later in this text the formulation of the hypothesis of rational expectations within the Arrow-Debreu contingent claim framework was carried out only in “Investment under Uncertainty” in a joint work of Lucas and Prescott in 1971. Yet this endeavor was only made possible given Lucas’ insistence in formulating business cycles models within a general equilibrium framework, according to which was not only an act of honoring one’s intellectual ancestor, but also a way to demonstrate the limits of interventionist policies brought about by the Keynesian revolution (Lucas, 1977). Thus, the importance of such framework is that:

(...) the contingent claim interpretation of a competitive equilibrium model considers all information to be simultaneously and freely available to all traders and many important results are crucially dependent on this assumption. It was soon recognized by many researchers that the idea of viewing a commodity as a function of stochastically determined shocks is an invaluable one also in situations in which information differs in various ways among traders. Indeed, it is this idea that permits one to use economic theory to make precise what one means by information, and to determine how it is valued economically. (Lucas 1980: 285)

Along the same line, Lucas (1980, p. 286) argues that: “It is now entirely practical to view price and quantity path that follow complicated stochastic processes as equilibrium “points” in an appropriately specified space.” In order to conclude this discussion, without implying the inexistence of controversies, equilibrium models of business cycles:
(...) are models that utilize the contingent claim point of view described in the last section in an essential way, and in which prices and quantities are taken to be always in equilibrium. In these models, the concepts of excess demands and supplies play no observational role and are identified with no observed magnitudes. In contrast to the static equilibrium models available in the 1940s, equilibrium models of this new class seem to do about as well in fitting time series as do models based on the neoclassical synthesis. (Lucas 1980: 287)

It is interesting to observe that although a complete general equilibrium framework would appear only with the paper published in 1972, the contingent claim formulation had already helped open the way for new understanding in the field of applied economics, especially those involving uncertainty.

Without advancing too much on the issue of the next section, I argue that it was only after the application of new tools related to dynamic programming, developed along with Prescott in a 1971 paper to deal with expectation, that a complete general equilibrium model was finally acquired. I will also argue in the final section that “Expectation and the Neutrality of Money” (1972) is the third in a series of three of Lucas’ papers which gave rise, some would argue directly or indirectly, to the dynamic stochastic general equilibrium (DSGE) model, a baseline model of today’s macroeconomics.


“Expectations and the Neutrality of Money” (1972) by Robert Lucas is arguably the first article to weave all the elements raised before into a single model: a representative agent with optimizing behavior, a mathematically proven general equilibrium framework and the modeling of rational expectations. In Chari’s (1998) words that paper “(...) is sometimes heralded as revolutionary, marking the beginning of the end of “Keynesian” economics and the birth of “rational expectations” economics.” The successful “artificial model economy” (Lucas, 1980) represented in that paper depended in large part upon the modeling of rational expectations, as mentioned before, carried out first in Lucas and Prescott (1971). In this section I describe briefly some historical aspect of the development of dynamic programming, a mathematical tool engendered during
world war II (WWII), and which consisted on the mathematical underpinning of Lucas’ theoretical advancement. Later, I attempt to describe the modeling of rational expectations in 1972s paper inherited from Lucas’ joint project with Edward Prescott.

By far the most important event to steer the massive use of applied mathematics, statistics and economics, in general, and of dynamic programming, in particular, was the need to provide the highest degree of efficiency required in times of war efforts\(^\text{10}\) in activities such as the allocation and stockpiling of resources (weapons, for example) and the minimization of time to accomplish various duties, a reasoning and a practice Judy Klein (2007) called “the science of economizing”. “The more the military could rely on mathematics to design mechanism and operations, the more they could save on time and scarce resources.” (Klein 2007: 5). A key figure in this undertaking was Richard Bellman, a mathematician working for the RAND corporation\(^\text{11}\) in the 1950s and 1960s with allocation models and control processes “(...) consistent with an economic way of thinking.” (Klein, 2007, p. 6). During the early 1950s, Bellman solved dynamic optimizations problems by way of functional equations he helped generalize. This research program is called dynamic programming. Judy Klein argues that:

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\text{(...) the context for Bellman’s first formulation of a dynamic programming protocol was the uniqueness of nuclear warfare, including decision-making with regard to strategic bombing and allocation of scarce fissionable material for atomic bombs. In the latter half of the 1950s, the major application of Bellman’s marriage of dynamic programming and control engineering was determining and maintaining optimal trajectories – particularly those formulated for a race of the two superpowers – the USA and the USSR – in outer space. (Klein 2007: 12)}
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\(^{10}\) It does not need to be necessarily a hot war like the WWII to provide the incentives to apply mathematical tools discussed above. The cold war between the U.S.A and the former USSR also worked as a substantial influence in the process. Indeed, most of the mathematical applications used took place following WWII.

\(^{11}\) Right after WWII, in March 1946, the US army air force began with the Douglas aircraft company a project, called project RAND, aimed at providing the military with researches in decision problems. The contributions would come from the field of applied mathematics, statistics and economics. The project became independent in may 1948 and has been operating under the name RAND CORPORATION ever since. It is still located in Santa Monica, California, United States, where it was born.
In fact, already in the 1950s, given Bellman’s exposure of his work in the meetings of the econometrics society, many of the mathematical models he helped create or improve were later used in microeconomics and in macroeconomics.

In the contexts of management science and operations research, microeconomists, usually with funding from the Office of Naval Research or the US Air Force’s contract with the RAND corporation, were the first economists to use Bellman’s generalized recursive optimization. As the space race heated up in the late 1950s, optimal control theory presented two approaches that economists could appropriate for their own theories on economic growth: Pontryagin’s elegant application of his maximum principle to the calculus of variations and Bellman’s heuristic application of his principle of optimality to dynamic programming. (Klein 2007: 43)

However:

It was not until after Robert Lucas and Edward Prescott’s 1971 Econometrica article on “Investment under Uncertainty” that macroeconomists began to systematically apply Bellman’s state-transition methods of recursive optimization to their theories of rational expectations, business cycle theory, and economic growth. (Klein 2007: 44)

The authors’ motivation in the paper was to broaden the existing empirical literature on the demand for investment goods by including a group of explanatory variables labeled “future demand” composed of sales, profits and stock price indexes. Given the uncertainty of such future demand, the firm is imbedded with a optimizing behavior that depends on the way it forms expectations. In practical terms it requires the modeler to formulate the problem by postulating some forecasting rule for firms.

Typically the forecasting rule postulated takes the form of anticipated prices being a fixed function of past prices – “adaptive expectations.” But it is clear that if the underlying disturbances (in our case, the demand shift) has a regular stochastic character (such as a Markov process), forecasting in this manner is adaptive only in a restrictive and not very interesting sense. Except for an unlikely coincidence, price forecasts and actual prices will have different probability distributions, and this difference will be persistence, costly to forecaster, and readily correctible. (Lucas and Prescott 1971: 660)
In an innovative solution the paper proposed “(...) go to the opposite extreme, assuming that the actual and anticipated prices have the same probability distribution, or that price expectations are rational.” (Lucas and Prescott 1971: 660)

In Lucas’ 1972 paper, referring to what he thought were his main contributions, he wrote:

The substantive results developed below are based on a concept of equilibrium search which is, I believe, new (although closely related to the principles underlying dynamic programming) and which may be of independent interest. In this paper, equilibrium prices and quantities will be characterized mathematically as functions defined on the space of possible states of the economy, which are in turn characterized as finite dimensional vectors. This characterization permits a treatment of the relation of information to expectations which is in some ways much more satisfactory than is possible with conventional adaptive expectations hypotheses. (Lucas 1972: 67)


5. A Summing Up

The description of a Lucas’ equilibrium account of the business cycle I have attempted here relied on the story of how he weaved three different economic concepts into a single model. In other words, I have tried to demonstrate how Robert Lucas Jr. has applied a wide variety of economic theories to the understanding of cyclical phenomena and ended up building a theoretical structure that interpreted them in an equilibratory sense. By attaining continuous market clearing results by way of modeling both the optimizing behavior of representative agents and rational expectations Lucas gave rise to a new school in macroeconomics denominated new classical school (in spite of the fact that Lucas
never saw any usefulness in applying such labels). It has been claimed in this paper, however, that the research program he helped inaugurate was not a smooth one in the sense he pieced together all necessary elements at once. In fact, from the time he tried to make Keynesian contributions in the 1960s to the point he became its fiercest critic Lucas scrutinized the most modern tools available in economics (the contingent-claim hypothesis) and combined it with others he rescued (microfoundation and dynamic programming) in a period that spans roughly fifteen years.

Did Lucas’ project represent scientific progress in economics, for example, in a inductivist sense described by Weintraub (2002)? It can be argued that much of Lucas’ work in the 1970s opened up the road for the advent of new macroeconomic schools like the Real Business Cycles and the New Keynesian in the 1980s. By late 1990s, Goodfriend and King (1997) coined the term “new neoclassical synthesis” to a movement many economists recognized as the theoretical and methodological fusion of the two schools aforementioned and by which the resulting theoretical body is a new class of models called dynamic stochastic general equilibrium (DSGE) models. Along this line “…if you tell a story that in a synthesis you merge the strengths of the schools that preceded it, how can one oppose the view that synthesis means progress?” (Duarte 2012: 219)

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12 After a period doing research in business cycles, Robert Lucas shifted his interest to economic growth in mid 1980s where he also made seminal contributions.
6. References


