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Coast to Coast: How MIT's students linked the Solow model and optimal growth theory

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Abstract:

Textbook narratives usually describe the Ramsey-Cass-Koopmans model of optimal growth as an important development over Solow's model. The constant saving rate rule of the latter is replaced by an endogenous determination of savings rates based on utility maximization behaviour of the former. However, neither Tjalling Koopmans nor David Cass were trying to upgrade Robert Solow's modelling of savings with their contributions. Koopmans was pushing for utilitarian analysis to study economic growth within the activity analysis community, with the help of Edmond Malinvaud. Cass and his colleagues at Stanford's graduate program were exploring the multiple applications of the maximum principle on economic growth models, influenced mostly by Hirofumi Uzawa. When the group of Stanford students graduated and moved to different departments, Karl Shell brought his interest on optimal growth and technical progress to the MIT. There, he organized a conference in 1965-66 that united his Stanford colleagues and MIT young scholars to discuss optimal growth theory. The conference represented the formation of a scientific community that consolidated optimal growth theory in the second half of the 1960 decade. The link between optimal growth theory and Solow's model was consolidated by this community of young students. Most of them were advised by Solow. This paper plans to shed some light on the importance of the MIT in the history of optimal growth theory and to show how the standard textbook narrative is a MIT-only story.

Keywords: Optimal growth; History of Economic Thought; David Cass; Robert Solow.

JEL Codes: B22; B23; B29.

Coast to coast: How MIT's students linked the Solow model and optimal growth theory[1]

1. Introduction

Contemporary textbooks on macroeconomics or modern economic growth theory usually reconstruct the development of early models in a linear and straightforward way. The Solow model, being a starting point of growth theory, gets a sophisticated version as the Ramsey-Cass-Koopmans model. The constant saving rate rule of the latter is replaced by an endogenous determination of savings rates based on utility maximization behaviour of the former. It is possible to find adaptations of this basic story in different textbooks. A representative example of this narrative appears in Acemoglu (2009, 145-147) textbook on modern economic growth:

The ultimate purpose [of the Ramsey-Cass-Koopmans model] is to enrich the basic Solow model by introducing well-defined household preferences and optimization, and in the process clarify the relationship between growth theory and general equilibrium theory. This will enable us to open the black box of savings and capital accumulation, turning these into forward looking investment decisions. It will also permit us to make welfare statements about whether the rate of growth of an economy is too slow, too fast, or just right from a welfare-maximizing (Pareto optimality) viewpoint.(...)

The Solow growth model is predicated on a constant saving rate. It would be more informative to specify the preference orderings of households (individuals), as in standard general equilibrium theory, and derive their decisions from these preferences.

As it is also pointed out in Assaf and Duarte (2017), this type of narrative presented in textbooks fail to offer a correct historical account as it organize past developments usually based on theoretical concerns. The objective of historical narratives in textbooks is more connected to present a linear and clean timeline of theory development than a rich and detailed history of science. To understand how these linear and clean narratives emerge in textbooks, and if they bear similarities to historical facts, is also an interesting issue to investigate for historians of economics. This is one of the main objectives of this paper.

It is important to stress that neither Tjalling Koopmans nor David Cass were trying to upgrade Robert Solow's modelling of savings with their contributions. A closer historical look shows that Koopmans was pushing for utilitarian analysis to study economic growth within the activity analysis community, in particular with the help of Edmond Malinvaud. Also, Cass and his colleagues at Stanford's graduate program were exploring the multiple applications of Pontryagin's maximum principle on economic growth models, influenced mostly by Hirofumi Uzawa. Although the contemporary literature merged their models into one "Ramsey-Cass-Koopmans" model, Koopmans and Cass contributions to economic growth theory in the 1960s had their differences, but neither was trying to answer questions raised by the Solow model. In the first section of this paper, we will explore the different scientific communities where Koopmans and Cass were active and how they were trying to clarify issues studied by these specific communities rather than just enhancing Solow's model.

Looking at the first textbooks on economic growth published at the end of the 1960 decade and the beginning of the 1970 decade, it is possible to observe that this direct linkage of the Solow model and optimal growth models was not obvious. For example, Burmeister and Dobell (1970), two economists who got their PhD at MIT in the 1960s at the height of the interest in optimal growth theory in that economics department, do not put forward this narrative. In their textbook, the first chapter is dedicated to the Solow model, described as a one-sector growth model. The second chapter present variations of the Solow model where technical progress is not constant across time. Optimal growth theory is presented just in the last chapter, as a normative growth theory. Previous chapters discuss models often neglected on contemporary

[1] This working paper is a preliminary draft. Comments are very welcome.

textbooks, as two-sector models of growth, turnpike theorems models, growth in a monetary economy and the Leontieff multisectoral model of growth. In the textbook, these other models are more connected to the Solow model than optimal growth theory is.

It is easier to see that Koopmans and Cass contributions were not dedicated to just endogenize the savings rate of the Solow model if we contrast their models with another one published during the same period by T. N. Srinivasan. Srinivasan was a PhD student at Yale University in the first half of the 1960s decade, studying under Koopmans. During this period, he also spent time at the Cowles Commission. His model, often neglected in textbooks or simple accounts of the historical development of growth theory, directly aims at the savings rate determination problem. Srinivasan writes that the purpose of his model is to treat the savings rate as a derived decision rather than a constant proportion of income. He does that in a two-sector model, what may help to understand why his model is often forgotten in modern textbooks. In the second section of the paper, we analyse Srinivasan contribution as a first link between the Solow model and optimal growth theory.

Srinivasan's choice to use a two-sector model is not made on a vacuum. Hirofumi Uzawa, working in Stanford in the first half of the 1960s decade, was working on this kind of growth models, trying to understand if the Solow model results would still apply in a two-sector model framework. Srinivasan had Uzawa's work as a benchmark to his own model of optimal savings. In Stanford, Uzawa supervised a group of PhD students interested in optimal growth theory and eager to apply new mathematical tools to economic modelling, in particular the maximum principle. David Cass was one of this students – his optimal growth model was first published in his PhD thesis. Karl Shell was a PhD student at Stanford in this period. Working with Arrow and Uzawa as supervisors, his research was aimed on understanding technical progress in an optimal growth framework.

When the group of Stanford students graduated and moved to different departments, Karl Shell brought his interest on optimal growth and technical progress to the MIT. There, he organized a conference in 1965-66 that united his Stanford colleagues and MIT young scholars to discuss optimal growth theory. The conference represented the formation of a scientific community that consolidated optimal growth theory in the second half of the 1960 decade. The link between optimal growth theory and the Solow model was consolidated by this community of young students. Most of them were advised by Solow. This paper plans to shed some light on the importance of the MIT in the history of optimal growth theory and to show how the standard textbook narrative is a MIT-only story.

2. "Ramsey-Cass-Koopmans" de-homogenized

In scientific literature, in particular in economics, it is common to designate canonical models and theorems after the scientists who pioneered research in the field. It is easy to find references to the "Arrow-Debreu model" or the "Sonnenschein–Mantel–Debreu theorem", among many other examples. This practice can be misleading from a historical point of view, since it tends to homogenize the individual contributions and motivations on the development of the work. For example, Düppe (2012) argues that even though Arrow and Debreu were working together on the proof of the existence of a general equilibrium, they did not share the same intellectual motivations on doing that, and they played different roles during the process of their work. Dimand and Spencer (2009) discuss the contribution of Trevor Swan to growth theory, showing the differences among his work and Solow's, even though sometimes the literature uses the epithet "Solow-Swan model".[2]

The same apply to the so-called "Ramsey-Cass-Koopmans" model. This denomination aggregate three very different economists, who have never published any work together, had very different

[2] It is possible to find the "Solow-Swan model" name, for example, in Barro and Sala-i-Martin (2004) and Aghion and Howitt (1998).

backgrounds, different images of the discipline, and, in the case of the first one, lived in a different period of time.

The differentiation of Frank P. Ramsey is more straightforward. A brilliant Cambridge mathematician and economist, Ramsey sadly died when he was just twenty-six years old in 1930, but not before he made very important contributions to mathematical economics and other fields of research. The "Ramsey" on the "Ramsey-Cass-Koopmans" title can be interpreted as a historical recognition of his pioneering work[3].

Tjalling C. Koopmans was born in the Netherlands in 1910. At age 17, he went to the University of Utrecht to study mathematics, with emphasis on analysis and geometry, "which were taught in a precise, but traditional style" (Lindbeck, 1992). Koopmans developed during this period a desire for a subject matter closer to real life than mathematics. In 1930, he switched his emphasis to theoretical physics, which he saw as a compromise between that desire and the need to work on a field in which his mathematical training could be useful (Koopmans, 1934, is an example of his work in the field of theoretical physics). Still in the thirties and in Utrecht, Koopmans met Jan Tinbergen, a leader in the field of mathematical economics in the Netherlands. Mathematical economics seemed like a better way of pursuing the understanding of real life problems with mathematical tools than theoretical physics, and Koopmans started reading Cassel and Wicksell on Tinbergen's advice. In 1940, Koopmans moved to the United States, where he worked on the war effort putting up a unified statement with information about British and American controlled ships. In 1942, still involved in the war effort, Koopmans produced a study of optimal routing of transportation (Koopmans, 1970, pp. 77-86). After the war, he joined the staff of the Cowles Commission for Research in Economics, invited by Jacob Marschak. In 1948, he succeeded Marschak as Director of Research in the institution.

As Director of Research, Koopmans took the initiative to organize in 1949 a conference on Activity Analysis of Production and Allocation. This conference came to be very influential in the transformation of economic science in the post-war period. It marked the emergence of a new scientific community making a new kind of economic theory growing from game theory, operations research and linear programming – even though many of the scientists in the community had no clear commitment to the discipline of economics, avoiding methodological problems extensively debated in the interwar period (Düppe and Weintraub, 2014). Koopmans' desire for solving real life problems with mathematics was met by the community pursue of a normative economic theory.

One of the fields that were studied by the activity analysis community was capital accumulation theory. Arguably, Malinvaud had done the most relevant work within the community in this field during the fifties. In Koopmans' book *Three essays on the state of economic science*, published in 1957 as a compendium of the theoretical developments made by the activity analysis scientific community, a long section of the first chapter was devoted to an exposition of Malinvaud's model of capital accumulation, published in 1953 in Econometrica. But, with the publication in 1958 of the book *Linear programming and Economic Analysis*, the focus of capital accumulation and growth theory done within this community shifted to the study of turnpike theorems, using versions of the Von Neumann's model (1945-6) as a framework. The bulk of the contributions on this literature was characterized by the usage of disaggregated capital, linear programming and absence of utility maximization. This last element did not please Koopmans and Malinvaud, who thought that as "economic organization aims at satisfying consumers' needs" (Malinvaud, 1953, p.241), a utilitarian criterion was necessary to assess efficiency of paths of economic growth. Koopmans (1964) classified Von Neumann's model as "poor economics", since it neglected consumption as an important variable. Koopmans contribution to optimal growth theory was presented in the Vatican Conference in 1963, at the height of this debate about the importance of utility and consumption in growth models. He was not trying to endogenize the savings rate of the Solow model, but rather constructing a model with a utilitarian criteria of optimality.

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[3] For a historical account of Ramsey life and contributions to economics, see Duarte (2009)

David Cass was born in 1937 in Hawaii, when Koopmans already was a promising young scholar in Utrecht. So in the sixties, when both published their contributions which would give their names to the canonical model, Koopmans was already an influential senior economist in an important scientific community, whereas Cass had just received his grad degree at Stanford University.

At Stanford, Cass became close to Karl Shell, his colleague as a grad student. Shell presented him to the Serra House, a small building in campus where economists and mathematicians interested in mathematical economics had their offices (Spear and Wright, 1998, p.535). Kenneth Arrow, Hirofumi Uzawa, Herbert Scarf and Samuel Karlin were some of the professors who had their offices at the Serra House. At the beginning of the 1960 decade, Uzawa was working on two-sector models of economic growth. He became Cass' dissertation advisor, and worked as a mentor to a group of grad students that were interested in growth theory, including Cass, Shell, Steven Goldman and Harl Ryder.

Uzawa's work in the beginning of the 1960 decade was dialoguing with the Solow model – but mostly about the one-sector assumption, and not the fixed savings rate. So if Cass' work was dialoguing with Uzawa's work, it is possible to see an indirect connection between the models. But it is clear that Cass motivation was not to endogenize the savings rate, as if his model was an improvement over the Solow model. In fact, Cass describes his motivation on dealing with optimal growth as more connected to applying Pontryagin's maximum principle to growth models than reaching a dialogue with the established literature:

Well, it was basically the fascination Uzawa had with the maximum principle. (...) Actually, even though Uzawa always went back and read literature and was always motivated by literature, I didn't pick that up from him at all; I just decided to work on this problem because the techniques were new and exciting and it seemed like an interesting problem. So I taught myself the maximum principle, some differential equations, and so on, by talking to people, seeing Uzawa working, and basically reading math books. Our bible at the time was Pontryagin's original book on the maximum principle. (Spear and Wright, 1998, p.537-539)

The three chapters of Cass' doctoral dissertation were dedicated to apply the maximum principle to growth theory – optimal growth, turnpike theorems (published in Econometrica in 1966), and two-sector models. Cass also thought his work had a different nature from the Solow model. His effort of modelling growth was connected to the formulation of a prescriptive theory of growth, whereas the Solow model was interpreted as a descriptive theory.

So here we have two very different scholars, from different generations, engaged in two different scientific communities, with different intellectual motivations. However, neither was trying to establish a dialogue with the Solow model. When Koopmans and Cass made their contributions to optimal growth theory, they weren't doing an effort to endogenize the savings rate of the Solow model, in a way to produce a better model of growth. In fact, this effort was being made at the same time by another economist, T. N. Srinivasan, a graduate student at Yale University.

3. The Srinivasan connection

Koopmans has described his time at Yale, after the reallocation of the Cowles Commission as a period where his research, "chiefly on optimum allocation over time, had more of a solitary character" (Lindbeck, 1992). However, it was at Yale that Koopmans worked together with the graduate student T. N. Srinivasan. During his PhD, Srinivasan developed a two-sector model of optimum economic growth that was dedicated to study capital accumulation within a framework where the savings rate would be given by a derived decision. Srinivasan thanks firstly to Koopmans for his collaboration and comments to the model produced, both in the working paper published by the Cowles Commission and in the paper published in 1964.

Srinivasan studied at Yale between 1958 and 1962, after becoming a bachelor in mathematics at the University of Madras, in India. After 1960, he begun to work in the Cowles Commission as a research assistant, when Koopmans became director of the institution again. He kept his interest in optimal economic growth theory during the sixties, publishing in 1969 a model of optimal growth under uncertainty with David Levhari. He was also interested in the theory of economic planning. In the seventies, Srinivasan was involved with economic planning in India, co-authoring books as *Scheduling the Operations of the Bhakra System* and *Optimum Requirements of Fertilisers for the Fifth Plan Period,* evidencing the normative nature of his research interests. In a sense, he shared with Koopmans the desire of solving real life issues with the mathematical tools that he got from his education, applying optimizing methods to economic planning.

The research done in Yale became the paper *Optimal Savings in a Two-Sector Model of Growth*, published in Econometrica in 1964. From a linear perspective of the development of growth theory, it is easier to connect the Solow model to the Ramsey-Cass-Koopmans model, since both models use a framework based on a one-good economy. However, as argued in the first section, neither Cass nor Koopmans had the aim to enhance the Solow model with their contributions. Although it is a two-sector model of growth, the main objective of Srinivasan's model is to open the black box of the savings rate. The reason why he chose to construct a two-sector model was the influence of Uzawa's work, in particular the papers published in 1961 and 1963. These motivations can be seen in the abstract of the 1964 paper:

In the recent literature[4] on mathematical models of economic growth, attention has been devoted mainly to the existence and stability of competitive equilibria. These models are based on a rather crucial but simple savings assumption: that savings form a constant proportion of income both being evaluated in terms of numeraire. In this paper, the savings decision is treated as a derived decision, i.e., as an implication of the more basic behaviour of utility maximization over time. Using a simple two-sector, two-commodity, two-factor model, optimal growth paths corresponding to the maximization of the sum of the discounted future stream of consumption per worker are worked out. Savings behaviour and asymptotic properties of these optimal paths for varying positive discount rates are also discussed. (Srinivasan, 1964)

The determination of the savings rate already was an important element of Uzawa's two-sector models. In the first version of his model, published in 1961 in *The Review of Economic Studies*, savings were determined by the production on the capital good sector. The underlying argument was that only the capitalists can save, whereas the working class would only consume all of their yield. This hypothesis was a consequence of Japanese Marxism influence on Uzawa, as he remembers in an interview (Okuno-Fujiwara and Shell, 2009, p.398). So in a sense, the savings rate of Uzawa's model is endogenous, since it is determined by an endogenous variable, the production of the capital good sector. However, the savings rate is still not determined by a utility maximizing behaviour, since there aren't any utility functions in the model. This way of modelling the savings rate was criticized by Solow (1961), in a comment about Uzawa's model. Solow argues that the stability of the equilibria is not guaranteed with a different hypothesis concerning the determination of the savings rate.

One of the objectives of Uzawa's second paper on a two-sector model of growth was to change the modelling of the savings rate, following Solow criticism. His model was still "an extension of the aggregate models of Solow and Swan" (Uzawa, 1963, p.105). In this model, Uzawa assumes that the savings rate is an increasing function of the interest rate and the income per capita. As in Srinivasan's model, the savings rate is not a constant, but again it is not derived from a utility maximizing behaviour. So the issue of how to model the savings rate of the economy arose in a two-sector model framework, and Srinivasan was engaged with this literature.

[4] The recent literature that Srinivasan is referring to are Uzawa's two-sector models of growth

Srinivasan's model is characterized by a two-sector economy, where the first sector produces a homogeneous capital good and the second one produces a homogeneous consumption good. Both production functions have constant returns to scale, with two factors of production represented by the capital good produced in the first sector and by labour, which grows exponentially with the population expansion. The consumption sector is more intensive in capital, a hypothesis needed to guarantee a sufficient condition for uniqueness of equilibria. This hypothesis appeared first in Uzawa's models. Also, there is no depreciation in Srinivasan's model.

Srinivasan constructs a central planner problem analogous to what would became a common structure in the optimal growth literature. The planner maximizes an integral that represents discounted paths of per capita consumption, with restrictions that guarantee the non-negativity of the variables, the feasibility of the factors of production and that the variation of the capital stock is equal to the output of the production function of the capital good sector, just as consumption must be equal to the production of the consumption good sector. The main results obtained by the model are that optimal paths of growth that correspond to different initial conditions of capital endowment, but that have the same intertemporal discount rate converge asymptotically to the same balanced growth path. The savings level, evaluated with the efficiency prices associated to optimal solutions, does not maintain a constant proportion of the income, diverging from Solow's hypothesis.

So from a historical perspective, the models who open the black box of savings and capital accumulation are in fact two-sector models. The Solow model motivated the two-sector model of Uzawa, not only to expand the aggregate model into a two-sector framework, but also to discuss the better way to model the savings rate. Those models were understood at the time as "descriptive" growth theory. On the other hand, Cass and Koopmans, when doing their models of optimal growth were doing "prescriptive" growth theory. Srinivasan's model is on the middle of the road. His model can be classified as optimal growth theory, but he was engaged in a debate with the descriptive growth literature. The model does not fit in the standard textbook narrative, where there is no distinction between "descriptive" and "prescriptive" and two-sector models rarely appear.

4. Moving from coast to coast: Karl Shell at Stanford and MIT

In the second half of the 1960 decade, the Stanford community interested in growth started to disintegrate, with Uzawa leaving the department to Chicago, and with his students graduating and assuming positions in different departments. Another department will raise as a dynamic centre of development of optimal growth theory, one that had already a strong tradition on the field, the MIT. An event that helps to understand this change, and the influence that the Stanford community exerted on this new MIT community is the arrival of Karl Shell in the MIT as a professor, and his four years working in the institution.

Shell arrived at Stanford with a bachelor in mathematics obtained at Princeton. There, he attended an economic theory course taught by William Baumol. It was at Baumol's classes that Shell had his first contact with the work of Arrow. Shell was so interested in Arrow's work that he decided to apply to Stanford because of him. At the time, Arrow was a professor in three different departments at Stanford, and it was Baumol who convinced Shell to apply to the economics department (Spear and Wright, 2001). At Stanford, Shell worked at the *Institute for Mathematical Studies in the Social Sciences*, located at the Serra House, where Arrow and Uzawa held their offices. In an interview, Shell describes his first year at Serra House and his first contact with growth theory through Arrow:

I participated from the start in the Serra House workshop on mathematical economics and econometrics run by Ken [Arrow] and Marc Nerlove. Manny [Menahem Yaari] and Ken-Ichi Inada were among the regular participants during my first year. Herb Scarf was an occasional participant. Hiro Uzawa was on leave at the CASBS [Center for Advanced Study in the Behavioral Sciences in Stanford, California].

(...) Manny presented some of his work on the consumption-loan model, my first exposure to OG[overlapping generations]. Marc presented a wide review of work on expectations, including a careful rendition of Muth's paper on rational expectations. This was before RE had become widely known. Ken gave a first draft of learning by doing. His seminar was my first exposure to modern growth theory, certainly my first exposure to endogenous technical change, and my first serious exposure to the calculus of variations (Spear and Wright, 2001, p.704)

Arrow played an important role in the optimal growth community in Stanford. Uzawa was appointed professor in the department mainly because of Arrow. He also made his contributions to the literature with a model that explored the consequences of the existence of learning by doing to technical progress and innovation. The model was published in 1962 in the Review of Economic Studies. Motivated by the empirical results found by Solow (1957) that show the impact of technical progress to economic growth and the fact that this important variable was exogenous in Solow (1956), what he saw as "hardly intellectually satisfactory" (Arrow, 1962b, p.155), Arrow sought to create an endogenous theory of changes in knowledge that could cause changes in production functions. Formally, Arrow modelled the learning process with a log-linear equation similar to what can be found in studies of learning processes made at Rand in the fifties. A key characteristic of Arrow's model is the combination of the existence of increasing returns in the production function with a competitive environment in a microeconomic level. The effects of learning by doing are externalities and therefore are not reflected in market prices.

Arrow planned to stay the academic year of 1963-64 abroad, visiting the Churchill College. Shell was invited to go, but he preferred to stay in Stanford. After that, Shell started to collaborate more with Uzawa and his students, Shell, Goldman and Ryder. According to Shell, it was with this group that he "learned two-sector growth, Pontryagin's maximum principle, and more" (Spear and Wright, 2001, p.705). He was jointly advised by Arrow and Uzawa, and his dissertation was based on extensions of Arrow's work on learning by doing and his contributions to inventive activity.

After graduating, Shell got his first job at the MIT, where he stayed from 1964 to 1968. His arrival at the department coincided with an increase of interest in growth theory among graduate students, especially in optimal growth theory. In those four years, Levhari(1964), Burmeister(1965), Sheshinski(1966), Stiglitz(1966), Nordhaus(1967) and Dixit(1968) published dissertations that investigated issues in growth theory. In the academic year of 1965-66, Shell organized at the MIT a conference on optimal growth theory that brought together his colleagues at Stanford and the emerging community of MIT's graduate students interested in growth theory.

In the preface to the volume where fifteen papers presented in the MIT conference were published, Shell writes that the major part of the works presented during the conference can be understood as elaborations and extensions of Ramsey's (1928) seminal contribution. Shell reads Ramsey in a way to connect his modelling style to the style practiced in the conference papers. To Shell, Ramsey's model had found a capital accumulation program that maximizes an integral of the utility generated by per capita consumption over time. This is a major problem faced by the models presented in the conference. However, Shell writes, the difference is that this new group of researchers have access to a mathematical tool unavailable to Ramsey in his time, Pontryagin's maximum principle. This tool could guarantee the necessary conditions to find an optimal path of economic growth. The conference assembled mostly the new generation of scholars interested in the optimal growth literature. The contributions given to the conference, published in the volume *Essays on the Theory of Optimal Economic Growth* (edited by Shell), are majorly made by young scholars who studied at the MIT or Stanford[5]. Therefore, the conference gives

[5] The list of contributors to the conference includes Paul Samuelson (MIT) and Karl Shell (MIT) with two papers each; Eytan Sheshinski (MIT), William Nordhaus (MIT), Harl Ryder (Brown University), Pranab Bardhan (MIT), Mrinal Datta-Chaudhuri (Indian Statistical Institute), Stephen Marglin (Harvard), Nicholas Carter (International Bank for Reconstruction and Development), Elizabeth Chase (MIT), Michael Bruno (MIT), David Cass (Cowles Foundation, Yale), Menahem Yaari (Cowles Foundation, Yale) and George Akerlof (Berkeley). In parenthesis are the institutions where each researcher was affiliated at

an outline of the transformation of growth theory in the second half of the 1960 decade, led by this emerging scientific community.

Analysing the references of each paper presented in the conference, it is possible to sketch a quick overview of this scientific community. It is also interesting to compare this conference with the one held in Cambridge (UK) in 1963 that assembled the activity analysis community to discuss models on growth and economic planning[6]. From the fifteen contributions published in the volume, fourteen were theoretical models. The only exception was an empirical exercise in economic planning, by Nicholas Carter. Compare it to the Cambridge Conference, where four of the eleven papers published were empirical investigations on economic planning. Frank Hahn (1968), in his review of the volume edited by Shell, notes this lack of interest on available instruments of planning on the MIT conference. He writes that "no one tells us how the economy is to be made to save the optimum amount each time, nor how we are to direct its investments in the right directions" (Hahn, 1968, p.563).

The two most referenced works in the conference are Cass (1965) and Pontryagin et al (1962). Among the fourteen theoretical papers, the latter is referenced in nine, while the former is referenced in six. Both of Samuelson's contributions do not cite these two works – they are preferred by the young scholars. Pontryagin's technique is the mathematical tool that unites the new generation of growth theory economists, and the blueprint for their models is given by Cass' contribution. As Duarte (2009, p.173) writes, "it is a very apt description of the optimal growth literature that emerged in the 1960s to say that it "carries the imprint: 'Economics by Ramsey; Mathematics by Pontryagin'" (Intriligator, 1969, p.117)".

The preference for Pontryagin's mathematics is pointed out by Hahn (1968). He writes that the contributions published – with some exceptions, he states – are fundamentally exercises in optimal control theory. Hahn criticizes the lack of discussion about the choice of the objective function to be maximized, what he perceives as an "unseemly haste to get down to the Hamiltonian" (Hahn, 1968, p.561). Hahn also points out that many interesting qualitative conclusions are related to the maximum principle and not from a thorough computation of the optimal paths. He also writes that, although the maximum principle is used in almost all papers, for many problems a more familiar "Kuhn Tucker" approach with transversality conditions could do the job.

Others economists that are remembered by a number of contributions of the conference are Koopmans, Ramsey and Solow. Koopmans' "On the concept of optimal growth", in its versions of 1963 or 1965, is cited four times, mostly because of its close association to Cass' work. Ramsey's 1928 paper is referenced three times by the young economists, but more as a historical reference that supports the importance of the work done. Solow's paper of 1956 is also referenced three times, as a seminal contribution to the theory of growth, thus creating a link between his work and optimal growth theory. Uzawa's two-sector models are referenced three times, the same as Ramsey and Solow. However, the three references to Uzawa's work are in Shell's two contributions and Ryder's paper, which were papers based on their PhD dissertations, with Uzawa as an influential advisor. Despite having contributed directly to the emergence of the optimal growth literature, Uzawa have not influenced much of the work produced by the MIT community[7]. His former student Cass has more influence in this emerging community – in fact, Uzawa's

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the time of the conference. It is interesting to see how the MIT aggregates a large number of researchers, while Stanford has lost its initial prominence, since Uzawa left to Chicago, and his students left to other departments (in the case of the participants on the conference: MIT, Yale and Brown)

[6] Some important members of the activity analysis community that contributed to the conference were Tjalling Koopmans, Edmond Malinvaud, Roy Radner, Lionel McKenzie, Robert Dorfman, Leonid Hurwicz, Sukhamoy Chakravarty and Maurice Allais. A group of planners also attended to the conference, as Salib Rafael from Egypt (who contributed with a paper on a model of investment planning), A. Nagy from Hungary and A. Nataf from France.

[7] This does not mean that Uzawa and two-sector models of growth were forgotten or neglected by the MIT students. For example, Burmeister's doctoral thesis was dedicated to study two-sector models of growth, and the textbook that he wrote with Dobell included chapters on two-sector models and turnpike theorems

papers published in the second half of the 1960 decade (see, for example, Uzawa, 1965, 1969) in growth theory resemble more Cass' style than the seminal two-sector models. Hahn (1968) argues briefly that in this community there is a view that two-sector models of growth "are probably not worth the effort" (p.564).

While Koopmans work is remembered by the researchers at the conference, there is no reference to the literature that he was engaging with his model. There is no citation of important contributors to growth theory within the activity analysis community, as Edmond Malinvaud, Roy Radner, Lionel McKenzie and Maurice Allais. These economists were participants in another conference dedicated to optimal growth theory that took place in Stanford in 1965. Boianovsky and Hoover (2014) describe in a footnote the encounter of two very different scientific communities in this conference:

Hahn was one of the participants at another conference (and monthlong workshop) on optimal growth, led by Arrow in Stanford in July 1965 (see Kenneth J. Arrow Papers, David M. Rubenstein Rare Book and Manuscript Library, Duke University, box 28; McKenzie 1999). Conference participants also included Lionel McKenzie, Roy Radner, Tjalling Koopmans, James Mirrlees, Uzawa, and Sheshinski (the only one from MIT), among others. Sheshinski presented a joint paper (with Burmeister) about reswitching. David Warsh (2006, 155–56) reports that optimal growth papers by some young MIT scholars were coldly received at the conference. Apparently, those papers were not included in the Stanford conference program, and were later discussed at the MIT 1965–66 seminar run by Shell. (Boianovsky and Hoover, 2014, p.213)

The MIT conference of 1965-66 marks the constitution of a new scientific community, represented by young economists, deeply influenced by Uzawa's students from Stanford, but centred in the MIT, where most of the young researchers were located at the time, mostly concluding their graduate studies. The agent that links Stanford and the MIT, representing this change of location of frontier scientific production in optimal growth theory, is Karl Shell. His four years at the MIT was the most productive period in growth theory in the graduate department. This change of location also represented a change of senior scholars considered as reference by the young researchers, with Uzawa losing some importance in favour of Robert Solow and his model, reinterpreted as a predecessor of optimal growth theory.

5. Solovian optimal growth

For a better understanding about how optimal growth theory has developed at the MIT in second half of the sixties, it is important to clarify the central role played by Robert Solow within this community. Solow completed his PhD at Harvard in 1951, with Wassily Leontief as his advisor. Solow started working at the MIT as a professor in 1949, thanks to a raising demand for professors of statistics and mathematics, both at undergraduate and graduate level. His model, published in 1956, was a perfect example of MIT's theoretical style. Solow presented his aggregate production function, identified a steady state that leaned on population growth and technical progress, and a convergence path defined by capital accumulation. More than that, his modelling style defined a literature and made canonical his contribution. A simple, clear and malleable artefact: a model seen as a tool, a drastic simplification of real world complexities, but not a negation of these complexities, and also not turning away from empirical motivations (Boianovsky and Hoover, 2014; Halsmayer, 2014). Solow was a central actor of the transformation of the MIT into one of the most important centres of economic theory in the post-WWII period. A way to assess his central role is to take a look at his increasing importance as an advisor of doctorate dissertations in the economics department. The following tables show Solow's role as an advisor in the decades of 1950 and 1960:

Table 1: MIT advisers and the number of students advised

MIT advisers	Number of MIT advisees	
	1950 decade	1960 decade
R. Solow	7	29
P. Samuelson	10	4
C. Myers	17	12
C. Kindleberger	13	25
M. Adelman	8	16

Source: Table adapted from Duarte (2014), p.89.

Table 2: MIT advisers and percentage of students advised

MIT advisers	Percentage of students advised	
	1950 decade	1960 decade
R. Solow	7,6	15,4
P. Samuelson	10,9	2,1
C. Myers	18,5	6,4
C. Kindleberger	14,1	13,3
M. Adelman	8,7	8,5

Source: Table adapted from Duarte (2014), p.87.

In the timespan between the publication of the Solow model and the literature of optimal growth theory produced by his advisees, the central role of the MIT on the production of growth theory was challenged. In the MIT, after the publication of the Solow model in 1956, growth theory continued to gain importance with the publication of *Linear programming and economic analysis*, that pushed the literature towards the turnpike theorems. However, it was not at the MIT that this literature developed the most, but in places like the Cowles Commission at Yale, under Koopmans. Then optimal growth theory started to build up at Stanford, with Arrow, Uzawa and their students. When Shell moved to the MIT and a group of students advised by Solow started a large production of models with the techniques first explored by Stanford students, the MIT comes back as the most important department doing growth theory in the 1960 decade. Other departments started to invite MIT students as visitants to enrich their own seminars on growth theory. Boianovsky and Hoover (2014, p.217) report an episode when Uzawa, who had recently transferred to the University of Chicago, brought MIT students to visit his new department:

Shell [Spear e Wright, 2001, p.709] recalls: "One summer (...) Hiro [Uzawa] borrowed a dozen students, taking them to Chicago." Although, Shell continues, "friendships and rivalries were strengthened in the Chicago heat," Uzawa wrote to Lionel McKenzie, "I am afraid [the Chicago 1965] seminar might not have had any particular impact upon the students who visited from MIT" (undated memo from Uzawa to McKenzie, Solow Papers, box 58). Underwriting this assessment, Uzawa observed, "I have been enjoying my association with the group of MIT students. We have been working on a couple of projects on *economic growth, although none of us have come up with anything concrete" (Uzawa to Solow, June 18, 1965, Solow Papers, box 61). But still, he was "particularly impressed by Joe Stiglitz."*

The students who were advised by Solow and Samuelson, after concluding their graduate studies, became professors in various departments, spreading MIT's modelling style to other important universities. Nordhaus became a professor at Yale in 1967, Burmeister took a position in Wharton in 1965, Akerlof went back to Berkeley, where he already had worked as an assistant professor even before starting his graduate studies at MIT. Stiglitz also moved to Yale in 1970. Shell left the MIT to the University of Pennsylvania in 1968, reuniting with Cass at the department. Levhari and Sheshinski moved to Israel, becoming professors at the Hebrew University (Blaug, 1986).

Besides working as an adviser, Solow also played an important role as a professor. Burmeister (2009, p. 35-38) presents an outlook of course 14.123 taught by Solow in the academic year of 1962-63 in advanced economic theory. The course was split with Ed Phelps, who was visiting MIT in that year, coming from Yale. David Levhari and Rodney Dobell (who co-authored with Burmeister one of the first textbooks on growth theory) were also attending class in the same year. Among the topics discussed during the course were classes dedicated to understand the meaning and how to study the growth of productivity and technical change, classes taught by Phelps about the golden rule of consumption, Arrow's learning-by-doing model and Pontryagin's maximum principle.

At the MIT, technical change would start to be formally modelled by visiting researchers in the department, Christian von Weizsäcker and Charles Kennedy. The former was a visitor during the academic year of 1962-63 and the latter in May 1964 (Boianovsky and Hoover, 2014, p.215). In a NBER publication in 1967, Solow discusses the recent developments of the theory of production, specifically about technical progress, sharing his views about the current state of this literature. Solow writes that although production theory is essentially a microeconomic theory, most literature devoted to this issue in the sixties was more connected to the field of macroeconomics. Because of that, to Solow the theory of production created during that period "merges imperceptibly with the theory of economic growth" (Solow, 1967, p.28).

Solow names one of the ways of modelling technical change as "factor-augmenting technical change". This kind of model can be found, for example, in Nordhaus research. The defining characteristic of factor-augmenting technical change models is to write a production function dependent to a vector of parameters – for example, $F(a(T)K, b(T)L)$. The factors a(T) and b(T) relate through an invention possibility frontier, that creates a tradeoff between the rate of growth of each factor. This method is used in the literature in Kennedy (1964), a model that the author developed during his visit at the MIT. Samuelson (1965) and Drandakis and Phelps (1966) also use this modelling tool to study induced bias in technical progress, i.e. if the economy tends to produce technical progress through a productivity increase in labour or capital. Nordhaus (Shell, 1967) presents a version of this problem, but with the modelling style of the young researchers of the MIT, with an optimization problem to choose the factors that raise productivity to determine the optimal rate and direction of technical progress. Nordhaus assumes that the central planner can allocate a fraction of the labour force to research to expand the invention possibility frontier in order to change the rate of technical progress. Maximizing an integral of discounted utility of per capita consumption, in the style of optimal growth literature, the planner can decide the optimal rate of technical progress.

Solow (1967) also discusses Arrow's learning by doing model (1962b) as a recent development of the theory of production. In the scientific community of young MIT researchers, the learning by doing model was explored by Levhari (1964) and Sheshinski (1966). Like in Arrow's model, the results show that the rate of economic growth is proportionate to the rate of population growth, since technical progress accelerate with an increase of the labour force of the economy. Although the results are qualitatively similar, Levhari and Sheshinski translated Arrow's model into the modelling style of optimal growth theory. Arrow's model studies optimal growth maximizing a discounted utility function of per capita consumption.

However, his model lacks a structure where Pontryagin's maximum principle can be used as a solution strategy, something that can be found, for example, in Sheshinski (Shell, 1967) model.

The activity analysis scientific community is also mentioned by Solow. Malinvaud (1967) presents the activity analysis approach as a criticism to the standard neoclassical modelling of production. Solow, however, writes that the results obtained by the contributions of this community just reinforce the results obtained by the neoclassical theory of production:

I take the development of linear theory to be a major advance in the theory of production (...)however the results of activity analysis and linear programming have gone to confirm and considerably to deepen the main insights of the smooth theory of production. They have not changed the results in any sharp or unpredictable way. (Solow, 1967, p.26)

With this reasoning, the models produced by the activity analysis community can be absorbed into the growth literature produced at the MIT. Since they do not provide any different results from the neoclassical theory, it is possible to neglect these developments in a cleaner and linear narrative, like those put forward in textbooks. However, the textbook account that links directly the Solow model with the Ramsey-Cass-Koopmans model is a contemporary narrative. It was not present in the first textbooks on growth theory, including those written by MIT alumni, like Burmeister and Dobell (1970) that it was already discussed in the introduction of this paper. Other books from the period like Stiglitz and Uzawa (1969) and Sen (1970) also do not put forward a linear narrative like contemporary textbooks.

This suggests that the appearance of the kind of narrative that merge optimal growth theory and the Solow model and excludes two-sector models and turnpike models from the development of growth theory is also due to posterior developments in growth theory after the 1970 decade. But looking at the history of growth theory in the 1960 decade shows that this narrative could be summed up only by the theory produced at the MIT.

6. Preliminary conclusions

This paper tries to construct a more nuanced narrative of the early developments of optimal growth theory, also pointing out that the linear narrative put forward by contemporary textbooks are a MIT-only story. In the first section, it was discussed how neither Koopmans nor Cass were trying to "enrich the basic Solow model by introducing well-defined household preferences and optimization" (to quote Acemoglu, 2008), but instead were engaging in discussions within their particular scientific communities. It was also discussed how the modelling of the savings decision in order to endogenize the determination of the savings rate was first discussed in a two-sector framework, by T. N. Srinivasan at Yale. Srinivasan work influenced both Koopmans and Cass contributions, and it was influenced mostly by Uzawa's two-sector models of growth. This kind of framework, though important at the time, do not receive much attention from contemporary textbooks.

Shell's move from Stanford to MIT was an important event to consolidate the importance of the latter department at the centre of optimal growth theory literature. It was at Stanford that a first group of young researchers, mostly influenced by Uzawa, created the framework of modern growth theory, "Pontryaginizing" optimal growth. But it was at the MIT that this literature was consolidated, with the conference organized by Shell at the MIT in 1965-66 representing the emergence of a new scientific community. The consolidation of optimal growth theory was mostly carried out by MIT students advised by Robert Solow, who was gaining importance at the department in the 1960 decade, particularly as an academic advisor.

Thus Solow, who was a key player in the "descriptive" theory of growth, also became a very important reference in the "prescriptive" theory literature, even though the development of optimal growth theory was not so connected to the literature of the Solow model. Mangling the descriptive and prescriptive theory of growth as contemporary textbooks do can then be understood as constructing an MIT-only story. Constructing a narrative where optimal growth theory emerges as a natural consequence of a sophistication of the Solow model put forward a very linear, progressive and simple account of scientific history. Since two-sector models and activity analysis do not play a relevant role in this narrative, the contributions coming from the MIT – from Solow and his advisees – are sufficient to cover the entire relation between the beginning of growth theory in the 1950 decade and optimal growth theory in the 1960 decade. This leaves out the intricate details of the development of a distinct way of modelling economic growth.

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