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Keywords: international reserves; sovereign debt; sovereign default; partial default; interest rate spread.

JEL Codes: E43; F31; F34; F41.

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1. INTRODUCTION

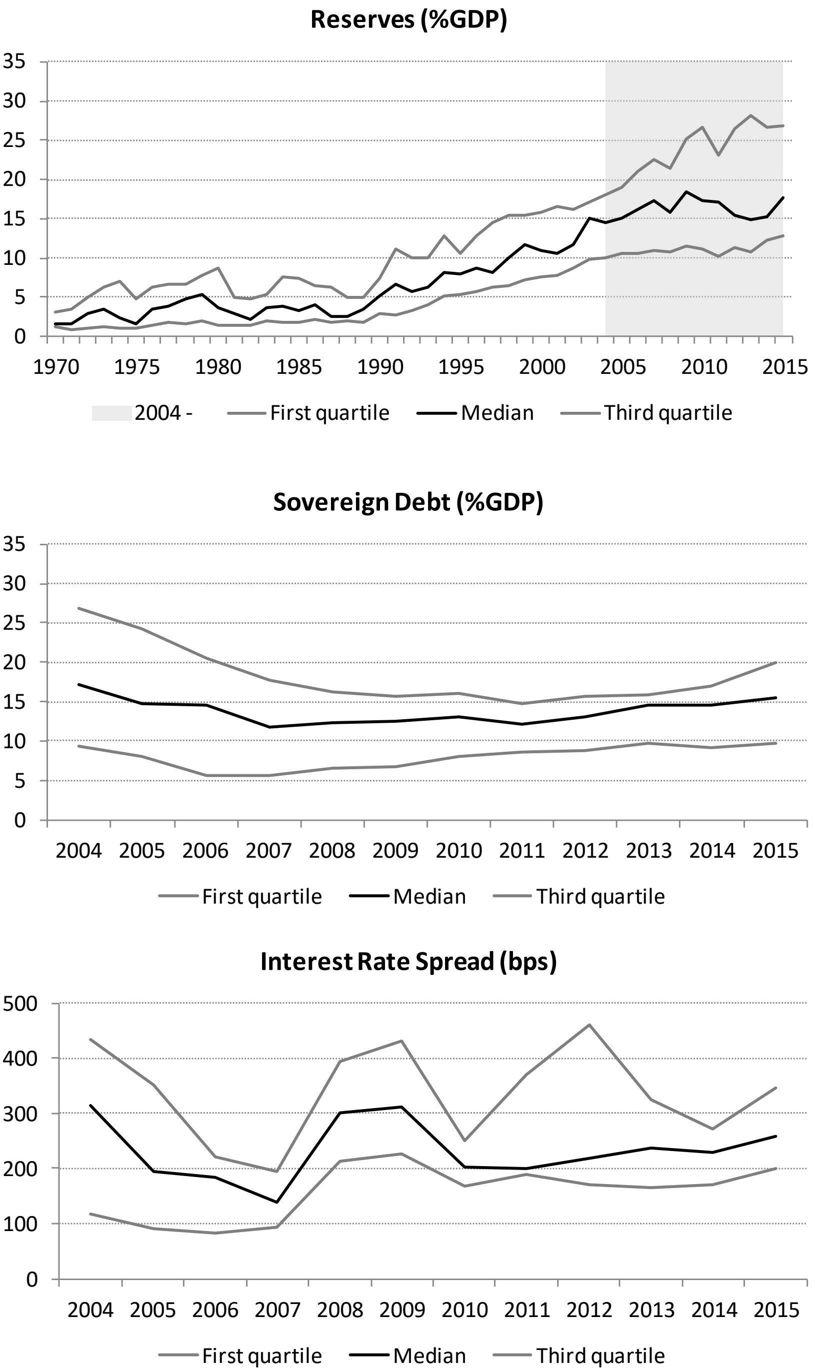
The amount of international reserves held by emerging countries in recent years is much higher than in previous decades (Figure 1). Currently, such governments also maintain positive quantities of sovereign debt[1] whose interest rates frequently exceed those earned on the international reserves by 200 basis points (Figure 1). Since governments could sell their reserves and reduce their indebtedness, the difference in yields makes the cost of keeping such stock of reserves meaningful (Rodrik, 2006).

In this paper, I investigate whether it is optimal for emerging markets to hold positive levels of both sovereign debt and foreign exchange reserves. To do so, I develop a quantitative model of strategic sovereign default in which debt, spreads, and reserves are endogenous. In this setting, international reserves are a tool to smooth consumption even after a delinquency. In this manner, I contribute to a vast literature that considers the recent build-up of international reserves as a form of precautionary savings to be used in moments of crises.

I extend the baseline model to incorporate partial debt repudiation, a feature present in the data (Cruces and Trebesch, 2013). I calibrate the model to mirror relevant characteristics of emerging market economies and quantitatively show that the optimal policy is to hold positive amounts of reserves. With risk-averse lenders, the model exhibits: i) average sovereign debt of 15.4% of GDP, ii) average spread of 242 bps, and iii) a ratio between volatilities of consumption and output of 0.97. Besides these targeted statistics, the model generates a stock of foreign exchange reserves of 7.7% of GDP, below the 16.4% observed in my sample of emerging markets between 2004 and 2015, but notably different from zero.

[1] Public debt owed to non-residents, issued abroad or at home.

Figure 1 – International Reserves, Sovereign Debt, and Spreads in Emerging Markets.



Note: The figure plots the median and the interquartile range for international reserves, sovereign debt and interest rate spreads for a balanced panel of 22 emerging countries. Foreign exchange reserve data come from the updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007). Sovereign debt is from Arslanalp and Tsuda (2014), includes foreign participation in local government debt markets, and starts in 2004. Spreads information comes from the Emerging Markets Bond Index Plus (EMBI+ blended). Countries in the sample are Argentina, Brazil, Chile, China, Colombia, Egypt, Hungary, India, Indonesia, Malaysia, Mexico, Peru, Philippines, Poland, Russia, South Africa, Turkey, Ukraine, Uruguay. The shaded area in the first panel represents the common sample to the three variables.

In the model with full debt repudiation, the value of defaulting is independent from the current debt. This happens because, after a temporary exclusion triggered by the default, the government returns to markets holding zero debt, regardless of the debt level existent in the moment of default. However, in a model with partial default, the value of defaulting decreases as debt raises. In this case, when the exclusion from credit markets finishes, the sovereign reentries the international debt market carrying a share of its previous liabilities. Thus, the inclusion of partial repudiation increases the incentives for repayment. Due to this mechanism, governments have more incentives to issue debt and accumulate reserves during good times (periods of high output), in line with the empirical evidence. Furthermore, the gathering of reserves during good times also generates a negative correlation between spreads and reserves, as in the data.

This paper relates to the literature that studies the simultaneous accumulation of sovereign debt and international reserves by emerging markets using quantitative models of default. Alfaro and Kanczuk (2005), Arellano (2008), and Aguiar and Gopinath (2006) developed this methodology based on the theoretical works of Grossman and van Huyck (1988) and Eaton and Gersovitz (1981)[2].

The first article to include the option to accumulate international reserves (a risk-free asset) in this setting is Alfaro and Kanczuk (2009). In their model, the only use of reserves is to smooth consumption, particularly after a default occurs and the economy is excluded from international financial markets. However, reserves are costly because their return is lower than the interest rate paid on sovereign debt. Such spread reflects the probability of default, a strategic choice by the local sovereign who cannot commit to honor its obligations. Thus, the local government chooses quantities of debt and reserves, and when to default. Alfaro and Kanczuk (2009) find that the optimal policy is not to hold reserves at all, despite their low cost (average spread of only 60 bps, in their benchmark calibration)[3]. Instead, they recommend that governments should use reserves to reduce their indebtedness.

[2] Recent surveys of this approach are Stahler (2013), Aguiar and Amador (2014), and Aguiar et al (2016).
[3] In their model, with full default, the average spread is approximately the same as the default frequency. Given the average stay in autarky of two years, it is possible to infer that 1.29% of time excluded from markets implies 0.65% of default frequency and similar spreads.

Salomão (2013) develops a model whose only difference from the one of Alfaro and Kanczuk (2009) lies in the functional form of the direct output cost of default[4]. Instead of proportional costs, she uses an asymmetric functional form proposed by Arellano (2008)[5]. In this case, costs are smaller when output is low. Her model presents positive average levels of debt and reserves, but mean spreads remain low, 60 bps. The shape of the direct cost of default matters, because with asymmetric costs the model produces higher average debt using a more patient domestic sovereign[6]. This agent perceives the cost of holding reserves (the interest rate spread) as lower, and chooses to accumulate more assets.

Alfaro and Kanczuk (2017) change the benchmark model turning it into a two-sector economy with traded and non-traded goods. They show that if sovereign debt is issued in local currency, a pattern observed recently in several emerging markets, it is possible to sustain positive levels of debt and international reserves even in an economy with proportional costs of default. Nevertheless, average spreads in simulated data remain low, 40 bps.

Bianchi, Hatchondo and Martinez (2018) obtain positive levels of both debt and reserves in a model with asymmetric costs of default[7] by changing the maturity of debt from short-term (one period bonds) to long-term (an infinite stream of coupons that decay at an exogenous rate). They also obtain average spreads of 240 bps, a value similar to the one observed in recent years in emerging markets. Their results are quantitatively more relevant when the economy faces rollover crises (exogenous increases in lender's risk aversion) and fiscal rigidity (a required fixed level of expenditure in a public good). Hernandez (2016) extends the model with long-term debt and investigates the role of reserves when the self-fulfilling rollover crises and multiple equilibria are possible.

I contribute to this literature by showing that the inclusion of partial default and risk-averse lenders in a model with short-term debt allows it to generate sensible levels of sovereign debt, spread,

[4] This extra cost, beyond exclusion from markets, is a common feature in this class of model and is necessary to induce positive levels of debt in equilibrium. See Aguiar and Amador (2014), and Phan (2017).
[5] Aguiar et al (2016) show that the assumption of proportional costs is better suited for a model in which output growth has a stochastic trend, as in Aguiar and Gopinath (2006). Assuming proportional costs and no stochastic trend for output growth, the model is unable to generate realistic levels of debt and spread/default frequency.
[6] The impatience is measured by the value of the domestic subjective discount factor, usually denoted in the macroeconomics literature by β. Alfaro and Kanczuk (2009) and Salomão (2013) use $\beta = 0.40$ and $\beta = 0.948$, respectively. In both cases, the international risk-free rate is 4%.
[7] They insert the immediate cost of default directly in the utility function.

and consumption volatility, and yet explain a large part of the international reserves holdings of emerging countries.

Other modeling approaches also highlight the role of international reserves as a precautionary savings mechanism. For investigations of the optimal level of international reserves in models with exogenous debt limits (or spread) and sudden stops, see Durdu, Mendoza, and Terrones (2009), Jeanne and Ranciere (2011), and Shousha (2017). Studies using the framework of Diamond and Dybvig (1983) include Aizenman and Lee (2007), Hur and Kondo (2016), and Corneli and Tarantino (2016). For an analysis of the relevance of the potential size of domestic financial fragility to explain observed levels of international reserves, see Obstfeld, Shambaugh, and Taylor (2010).

Dooley et al (2004) present an alternative view on the accumulation of reserves by emerging markets. They suggest that the build-up of reserves derives from a mercantilist policy to increase net exports by devaluating the domestic currency. Korinek and Servén (2016) formalize this idea in a model in which the accumulation of reserves undervalues the real exchange rate and stimulates the production of tradable goods, a sector with learning-by-investing externalities.

Gosh et al (2016), Obstfeld, Shambaugh, and Taylor (2010), and Aizenman and Lee (2007) provide empirical evidence on the determinants of the size of reserve holdings and compare the precautionary and mercantilist views.

2. MODEL

I model a dynamic small open economy in which the benevolent central planner receives a stochastic endowment every period. This agent issues only non-state-contingent debt, bought by foreign lenders, and buys a risk-free asset (international reserves). Since the sovereign lacks commitment to repay, every period it chooses whether to default on the stock of debt. In case of default, the sovereign is excluded from international markets by a random number of periods and faces a direct output cost. As default is partial, the new stock of debt upon reentry in the credit market is a share of the one defaulted upon.

Consider a representative agent whose preferences are given by equation (1), in which E denotes the expectation operator, c_t is the consumption of goods in period t, β is the domestic subjective discount factor, and σ is the coefficient of constant relative risk aversion.

$$U = E\left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}\right] \tag{1}$$

The endowment of the single good available in the economy, y_t, follows the autoregressive process described in equation (2) with ε_t representing a white noise with standard normal distribution.

$$\ln(y_t) = \rho \ln(y_{t-1}) + \eta\varepsilon_t \tag{2}$$

If the government chooses to honor its current obligations, it faces the budget constraint (3), in which q_t is the price of a one-period bond. This security pays one unit of the single good in the next period if the government chooses not to default. The planner can increase consumption borrowing from foreigners by issuing debt, d_{t+1}, or depleting the current stock of international reserves, a_t, whose constant price is q^a.

$$c_t = y_t + q_t d_{t+1} - d_t - q^a a_{t+1} + a_t \tag{3}$$

If the government decides to default, expression (4) presents its budget constraint. It expresses that the planner can still use and buy reserves, but cannot issue new debt. Beyond exclusion from international bond markets for a random number of periods, the domestic economy also faces a direct output cost after default. I use the specification in equation (5), proposed by Arellano (2008), frequently used in this literature, and consistent with the empirical evidence. This asymmetric function means that there are no direct costs of default up to a certain threshold (ψ), but they become positive beyond that point. Since sovereign defaults are associated with disruptions in the domestic financial market and credit is an essential input, this functional form captures the idea that output cannot be high even under a good productivity shock[8].

$$c_t = y_t^a - q^a a_{t+1} + a_t \tag{4}$$

$$y_t^a = \begin{cases} y_t, if\ y_t \leq \psi \\ \psi,\ if\ y_t > \psi \end{cases} \tag{5}$$

Now I write the sovereign problem in recursive form to understand the role of partial default. As usual in the literature, variables with apostrophe represent values at $t + 1$. For the value functions and restrictions defined below, I obtain policy functions for default (f), debt issuance (d'), and asset acquisition and consumption under repayment (a_R', c_R) and default (a_D', c_D).

Every period the sovereign decides to default or repay according to equation (6),

$$v(y, d, a) = \max_{f \in \{0,1\}} \{ (1 - f) v^R(y, d, a) + f v^D(y, d, a) \} , \tag{6}$$

in which the value of repaying is expressed by

$$v^R(y, d, a) = \max_{c_R, d', \mathrm{a}_\mathrm{R}'} \{ u(c) + \beta E_y [v(y', d', \mathrm{a}_\mathrm{R}')] \} , \tag{7}$$

subject to (3), $d' > 0$, and $\mathrm{a}_\mathrm{R}' > 0$,

and the value of defaulting is given by

$$v^D(y, d, a) = \max_{c_D, \mathrm{a}_\mathrm{D}'} \{ u(c) + \beta E_y [\theta v(y', \lambda d, \mathrm{a}_\mathrm{D}') + (1 - \theta) v^D(y', d, \mathrm{a}_\mathrm{D}')] , \tag{8}$$

subject to (4), (5) and $\mathrm{a}_\mathrm{D}' > 0$.

[8] See Mendoza and Yue (2012) for a general equilibrium model of sovereign defaults and business cycles that generates non-linear output costs. The asymmetry happens due to working capital financing constraints for imported inputs that lack perfect domestic substitutes.

In the previous equation the parameter θ measures the exogenous probability of regaining access to the international markets with debt level equal to λd. This modeling choice for partial default is similar to the ones used by Önder and Sunel (2016) and Hur, Kondo and Perri (2017). Nonetheless, I extend it to incorporate the presence of the risk-free asset. Hence, the value of defaulting depends on the current debt level due to the existence of partial default.

The price of international reserves, given by equation (9), is constant and depends only on the risk-free rate, r^*. Meanwhile, the price of debt reflects the sovereign's incentives to repay as perceived by risk-averse foreign lenders. They price the bond's payoff using the reduced form stochastic discount factor in equation (10). Arellano and Ramanarayanan (2012) and Bianchi, Hatchondo and Martinez (2018) use this specification in their quantitative models of sovereign default. In expression (10), the parameter κ dictates the risk premium and its correlation with the stochastic process for y_t. While $\kappa = 0$ leads to risk neutral lenders, positive values imply that lenders value more returns in states with negative income shocks, when default is more likely to happen.

$$q^a = exp(-r^*) \tag{9}$$

$$m_{t+1} = exp(-r^* - \kappa\eta\varepsilon_{t+1} - 0.5\kappa^2\eta^2) \tag{10}$$

Due to partial default, the price of sovereign bonds, q, depends on its own price during the exclusion from capital markets, q^D. Let $s = (y, d, a)$, $s_\lambda = (y, \lambda d, a)$ and E_y denote the conditional expectations operator. Then, equations (11) and (12) present the respective prices.

The price of debt depends on the current endowment, which brings information about its next realization, and on the future values of debt and reserves. Quantities of assets and liabilities in the following period are the relevant information for the lenders, because that is when the sovereign decides to repay or not. If the sovereign chooses to honor its obligations, the lender receives one unit of the good. In case of delinquency, the creditor holds a bond worth $q^d(y', d'', a'')$.

$$q(y, d', a') = E_y\{m_{t+1}[(1 - f(s') + f(s')q^d(y', d'', a'')]\}, \tag{11}$$

with:

$$a'' = a'_D(y', d', a'),$$
$$d'' = d'.$$

During the exclusion from markets, the price also hinges on the current endowment and on the future values of debt and reserves. If the exogenous exclusion from markets remains for one more period, bonds are priced $q^d(y', d'', a''_1)$. On the other hand, if exclusion ends, the recovery rate λ is applied and there are two possibilities: the government defaults again, and bonds are worth $q^d(y', \lambda d'', a''_2)$, or repays.

$$q^d(y, d', a') = E_y\left\{m_{t+1}\left[(1-\theta)q^d(y', d'', a''_1) + \theta\lambda\left(1 - f(s'_\lambda) + f(s'_\lambda)q^d(y', \lambda d'', a''_2)\right)\right]\right\}, \quad (12)$$

with:

$$a''_1 = a'_D(y', d', a'),$$
$$a''_2 = a'_D(y', \lambda d', a'),$$
$$d'' = d'.$$

The model represents a dynamic game played between a discretionary sovereign against a continuum of small identical foreign lenders. Given the lack of commitment, I focus on Markov Perfect Equilibrium.

Definition. A Markov perfect equilibrium is defined by:

i) A set of value functions $v(s), v^R(s), v^D(s)$ defined above.

ii) Policy functions $f(s)$, $d'(s)$, a'_R(s) and a'_D(s), and c_R(s) and c_D(s).

iii) Bond price functions $q(y, d', a')$ and $q^d(y, d', a')$.

such that

I) Given bond prices, the policy functions solve the Bellman equations (6) - (8).

II) Given the policy functions, the bond prices satisfy equations (11) - (12).

3. Functional forms and calibration

Table 1 presents the benchmark values for the parameters in the model. As a period in the model refers to one year, I use $r^* = 0.04$, a standard choice. The probability of redemption after default, θ, is 50%, entailing an average stay in autarky for two years, in line with estimates by Gelos, Sahay and Sandleris (2011). The recovery rate, λ, matches the complement of the average haircut (excluding highly indebted poor countries) estimated by Cruces and Trebesch (2013), 29.7%, considering 157 debt restructurings from 1978 to 2010.

For the endowment process, the parameters ρ and η are the same used by Alfaro and Kanczuk (2009), who obtained them from GDP data for a sample of emerging markets. These values are very close to the more recent estimates of Uribe and Schimitt-Grohé (2017). In order to discretize this process, I use the simulation method proposed by Schimitt-Grohé and Uribe (2009).

I calibrate the remaining four parameters $(\sigma, \beta, \psi, \kappa)$ to match four targets in the data: i) average sovereign debt of 14.1% of GDP[9]; ii) average interest rate spread of 234 bps; iii) 35% of this spread related to risk premium, and the remaining reflecting default probability; and iv) a ratio of 0.98 between volatilities of consumption and GDP. While the first two targets reflect the data in figure 1, the decomposition of total spreads between its two components and the volatility ratio come from Longstaff et al (2011) and Uribe and Schimitt-Grohé (2017) respectively.

I obtain a domestic discount rate, $\beta = 0.905$, similar to the values of Bianchi, Hatchondo and Martinez (2018), and Hernandez (2016). The resulting direct output cost of default is $\psi = 0.86$. Such parameters are mainly relevant for the first two targets: average debt and spreads. The value of the pricing kernel parameter, $\kappa = 7$, is the main determinant of the shares of the total spread associated with default risk and risk premium.

The risk aversion coefficient achieved is $\sigma = 5$. Du, Pflueger and Schreger (2017) set $\sigma = 10$ in a model of the currency composition of sovereign debt. This last figure is at the upper end of values

[9] Similar values are used by other studies of sovereign debt, as Hernandez (2016), Ottonello and Perez (2016), and Du, Pflueger and Schreger (2017).

considered plausible by Mehra and Prescott (1985) and within the range of estimates by Bliss and Panigirtzoglou (2004) and from other studies they summarize.

The model is solved numerically using value function iteration in a discrete state space. As suggested by Hatchondo, Martinez and Sapriza (2010), I find the equilibrium by solving the limit of the equivalent finite-horizon version of the model.

Table 1 – Parameter values

Parameter	Description	Value
σ	Domestic Risk Aversion	5
β	Domestic discount factor	0.905
ψ	Direct output cost of default	0.86
κ	Pricing kernel parameter	7
θ	Probability of re-entry after default	0.5
r*	Risk free rate	0.04
ρ	GDP persistence	0.85
η	Std. Deviation of innovation to GDP	0.044
λ	Recovery rate	0.7

4. RESULTS

Alfaro and Kanczuk (2009) point that reserve holdings reduce the cost of exclusion from capital markets and increase the temptation to repudiate debt. On the other hand, reserves are an option to avoid the costly tool of default and might contribute to debt sustainability. The default policy function for the benchmark calibration, depicted in figure 2, shows that the existence of a stock of reserves increases the amount of sustainable debt for a given level of output, opposite to the result of Alfaro and Kanczuk (2009). In the same direction, price functions in figure 3 indicate lower spreads (higher prices) when the sovereign decides to accumulate more assets for a given debt level[10], in line with the empirical evidence (Henao-Arbelaez and Sobrinho, 2017).

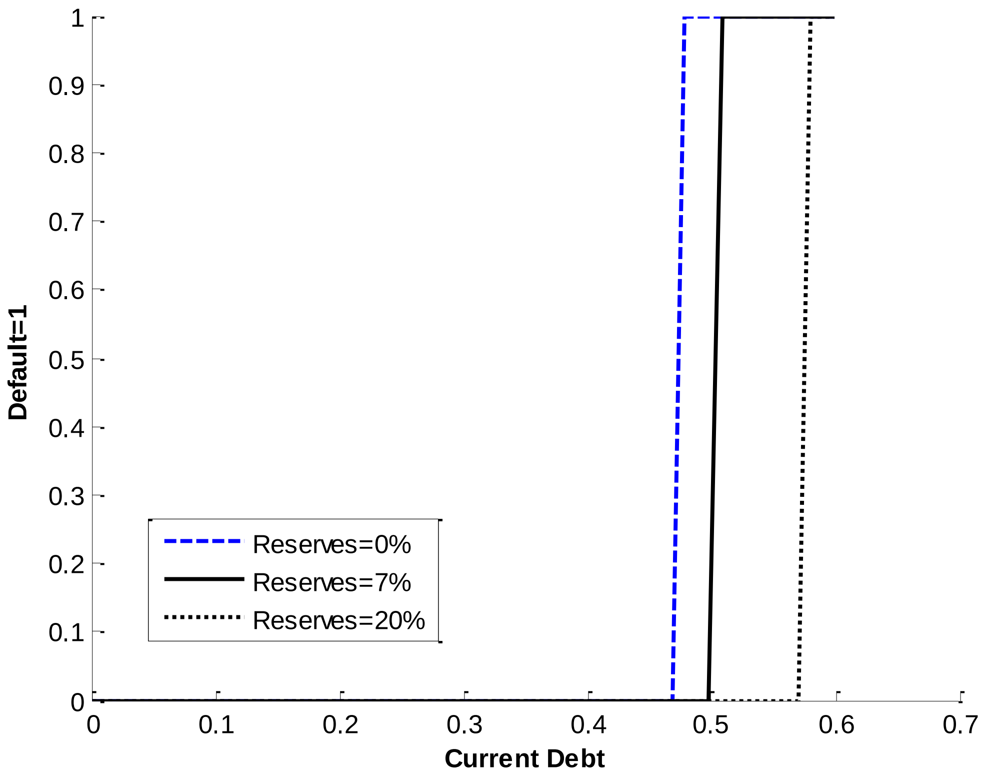
Partial default plays a relevant role in this result by allowing the model to achieve the desired debt level with a more patient sovereign (higher β). If I solve the model setting $\lambda = 0$, full debt

[10] In the model of Hernandez (2016), the sovereign can increase the amounts of both reserves and debt, keeping a fixed net position, and still face lower spreads. This happens due to the role of reserves in avoiding self-fulfilling rollover crises.

repudiation, and targeting the same average debt (therefore, changing the value of β), I obtain a result similar to that of Alfaro and Kanczuk (2009): reserves decrease debt sustainability. If I fix $\beta = 0.905$ and use $\lambda = 0$, the current quantity of reserves do not influence debt sustainability; the default policy function for the median output level is the same for different amounts of assets. In this case ($\beta = 0.905$ and $\lambda = 0$), the model generates a lower average debt level (5.5% of GDP).

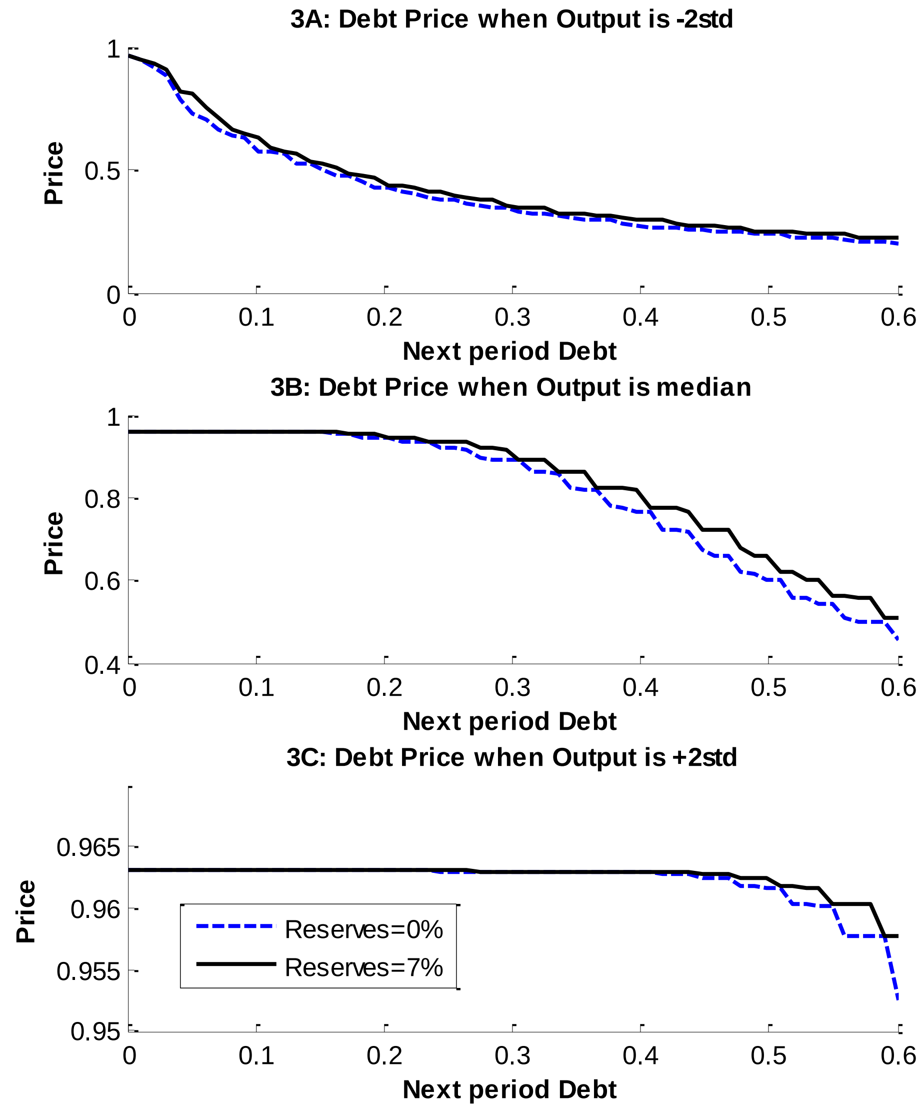
In the traditional model with complete default, the value of repayment (v^R) decreases with the debt level, but the value of default (v^D) is constant. Figure 4 shows that, due to partial repudiation, the value of default also falls as debt escalates, increasing debt sustainability. This creates an incentive for the joint accumulation of reserves and debt.

Figure 2 – Default Policy Function for the Median Output Level



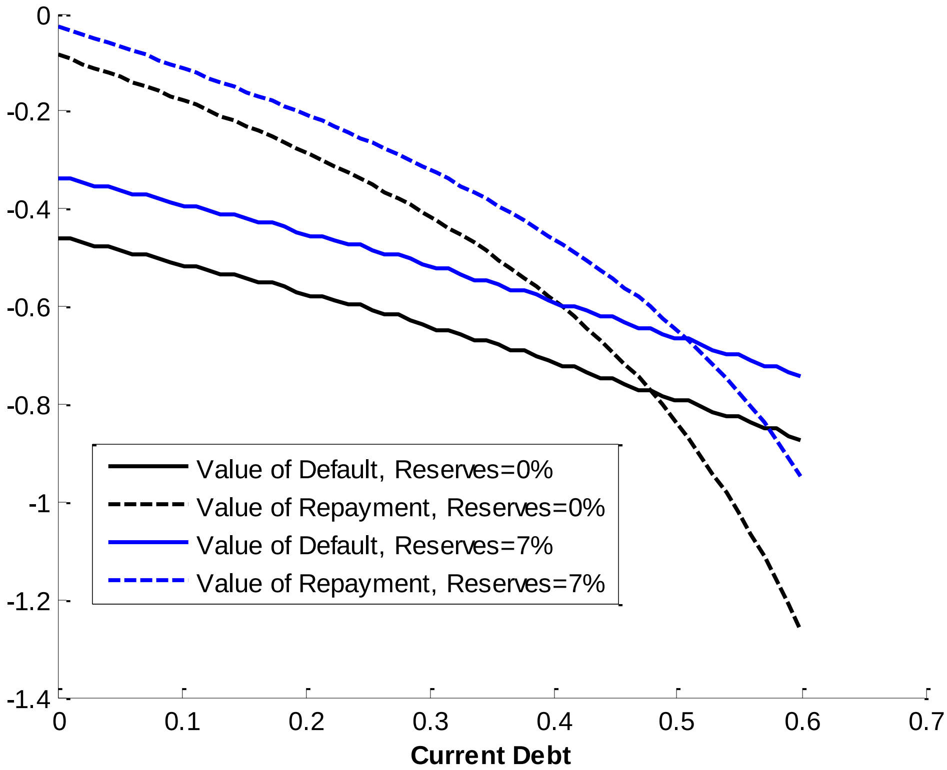
Note: This figure plots the default policy function for the median level of output. When the optimal choice is to default, the policy function is one. The horizontal axis represents current debt level in relation to the median output. Each line represents the policy function for a different level of reserves measured as a share of median output.

Figure 3 – Bond Price Function for Different Output Levels



Note: This figure plots the bond price function for three different levels of output: the median and plus or minus two standard deviations. The horizontal axis represents the choice of next period debt in relation to the median output. Each line represents the price function for a different choice of reserves level in the next period, measured as a share of the median output.

Figure 4 – Value Functions for Default and Repayment for the Median Output Level



Note: This figure plots the value functions for default (solid line) and repayment (dashed line) for the median output level. The horizontal axis represents current debt level in relation to the median output. Each color (for a pair of lines) represents the value functions for a different level of reserves, measured as a share of median output.

Table 2 reports basic statistics in the data and in model simulations. The benchmark model, presented in column 2, matches the four targeted statistics and produces average reserves of 7.7% of GDP. This number is below the observed in emerging markets since 2004, but close to the results of other papers in the literature, between 3% and 6%. This difference leaves room for alternative explanations for the recent surge in reserves, seeing that in this model reserves are useful only to smooth consumption. Positive correlations between reserves and both debt and GDP arise because during good times (high output) governments issue debt to accumulate reserves, in line with figure 4. Interest rate spread is counter-cyclical and negatively correlated with reserves[11].

[11] In a panel of 22 countries, Bianchi, Hatchondo and Martinez (2018) also find: i) negative correlation between debt (or reserves) and spread, and ii) i) positive correlation between debt (or reserves) and GDP growth.

Table 2 – Basic Statistics: Model and Data

Variables	Data 2004-15	Models: Benchmark	Debt only	Risk Neutral	Full default	Total Ext. Debt
	1	2	3	4	5	6
			Average			
Default frequency	--	3.8	3.9	6.1	0.7	3.8
Debt/GDP	14.1	15.4	9.5	20.4	5.5	33.0
Spread	234	242	248	189	164	229
Risk Neutral Spread	152	148	152	189	88	142
Reserves/GDP	16.4	7.7	--	3.3	5.1	5.4
			Standard deviation			
Consumption	0.98	0.97	0.99	0.96	0.97	1.05
Debt/GDP	4.4	8.2	3.8	6.0	4.6	7.7
Spread	85	540	551	376	336	499
Reserves/GDP	3.7	12.4	--	7.3	8.9	10.0
			Correlation with GDP			
Debt	0.0	0.5	-0.6	-0.2	0.6	0.5
Spread	-0.6	-0.6	-0.6	-0.6	-0.6	-0.7
Reserves	0.4	0.6	--	0.5	0.6	0.6
			Other correlations			
Debt & Reserves	0.3	0.8	--	0.2	0.8	0.9
Debt & Spread	-0.1	-0.2	0.3	0.2	-0.3	-0.1
Spread & Reserves	-0.4	-0.3	--	-0.2	-0.3	-0.2

Note: Column 1 presents basic statistics for emerging countries using data from figure 1. Each column from 2 to 6 brings statistics calculated from simulated data (500,000 observations) of a different model. See the main text for the calibration used in each column. Debt and reserves ratios to GDP appear as percentage points and spreads as basis points. Standard deviation for consumption reported relative to that of output. In column 1, growth rates are used to calculate correlations, except for spreads.

The benchmark model does not work so well in replicating volatilities, except the targeted one. The standard deviation of the spread of 85 bps – the median in the sample of 19 countries for the period 2004-2015 – is low in historical terms. Even extending the initial period of the sample, the standard deviation increases only to 160 bps. The only countries with standard deviation of the spread higher than the generated by the model, 551 bps, are Argentina (1620 bps), Russia (907 bps) and Ukraine (633 bps). The next one is Brazil with 353 bps[12]. The model also overstates the volatilities of sovereign debt and international reserves, and by a magnitude similar to the one identified by Shousha (2017) in a framework with exogenous spreads, financial frictions and sudden stops[13]. Nevertheless, the excessive model volatility might be reconciled with the data if the decade under investigation is considered as a sequence of good output realizations leading to low spreads, and high debt and reserves stocks with low volatility. Corroborating this interpretation, using data since 1970, the standard deviation of reserve holdings and total external debt[14], both as share of GDP, increase from 3.7% to 7.1% and from 6.9% to 16.1% respectively.

Comparing the benchmark model with the one in column 3, in which the government cannot buy assets, I highlight two main differences. The first is that in the "debt only" model the average sovereign debt is 9.5% of output, lower than 15.4% in the benchmark. It follows that when governments have access to risk-free assets they choose to accumulate more debt simultaneously. The second distinction is the sign of the correlation between debt and spreads. This correlation is negative, as in the data, only in the benchmark model. In this situation, the sovereign has more incentives to accumulate debt and reserves jointly in periods of elevated output, when spreads are low.

In column 4 of Table 2, I present results from a model in which lenders are risk-neutral ($\kappa = 0$) and the other parameters remain the same as in the model of column 1. Compared to the benchmark, average indebtedness rises, mean and volatility of the interest rate spread decrease[15], and consumption volatility continues unaltered. The optimal accumulation of reserves diminishes, but remains positive and in the range of results from other papers in the literature (3% to 6%). In this setting, reserves are still pro-cyclical and positively correlated with debt and negatively with spreads. These results indicate that the presence of risk-averse lenders increase the average level of reserves due to an amplification of

[12] See Aguiar et al (2016) for a discussion of the ability of this type of model to match spread volatility and the peculiarity of the Argentinean case studied in Arellano (2008), in which observed and simulated spreads are 544 bps and 636 bps respectively.
[13] None of the other papers investigating reserve accumulation using quantitative models of sovereign default reports these statistics.
[14] In this exercise I use total external debt, because sovereign debt data from Arslanalp and Tsuda (2014), including foreign participation in local markets, starts in 2004.
[15] Nevertheless, average spread is still higher than 60 bps, the value in the papers of Alfaro and Kanczuk (2017) and Salomão (2013).

the precautionary motive. With risk-averse lenders, spreads rise more during bad times. Not only the default risk grows, but also the premium charged by creditors. In this environment, foreign exchange reserves become an even more attractive form of insurance.

Data from a model with full default (zero recovery rate) and the same calibration of the benchmark model for other parameters appear in column 5 of Table 2. The model does not deliver a sensible debt level. The mean stock of reserves decreases, despite the lower spread, because the level of debt to be insured is smaller. Correlations do not change.

Table 3 – Basic Statistics: Model and Data

Variables	Data	Models				
	2004-15	Benchmark	g=0.12	σ=3.3, recalibrate	σ=2	σ=2, recalibrate
	1	2	3	4	5	6
			Average			
Default frequency	--	3.8	3.8	3.6	2.6	4.1
Debt/GDP	14.1	15.4	14.6	14.6	19.9	15.6
Spread	234	242	232	214	205	216
Risk Neutral Spread	152	148	143	148	116	171
Reserves/GDP	16.4	7.7	8.9	5.7	2.1	2.4
			Standard deviation			
Consumption/GDP	0.98	0.97	0.95	0.99	1.14	1.07
Debt/GDP	4.4	8.2	7.9	8.1	9.7	8.1
Spread	85	540	554	440	299	381
Reserves/GDP	3.7	12.4	13.0	10.9	5.9	6.7
			Correlation with GDP			
Debt	0.0	0.5	0.5	0.6	0.7	0.7
Spread	-0.6	-0.6	-0.6	-0.6	-0.7	-0.6
Reserves	0.4	0.6	0.7	0.6	0.5	0.5
			Other correlations			
Debt & Reserves	0.3	0.8	0.7	0.8	0.8	0.8
Debt & Spread	-0.1	-0.2	-0.2	-0.2	-0.3	-0.2
Spread & Reserves	-0.4	-0.3	-0.3	-0.3	-0.2	-0.2

Note: Column 1 presents basic statistics for emerging countries using data from figure 1. Each column from 2 to 6 brings statistics calculated from simulated data (500,000 observations) of a different model. See the main text for the calibration used in each column. Debt and reserves ratios to GDP appear as percentage points and spreads as basis points. Standard deviation for consumption reported relative to that of output. In column 1, growth rates are used to calculate correlations, except for spreads.

In column 6, I recalibrate the model with $\beta = 0.78$, $\sigma = 10$, $\psi = 0.82$ in order to achieve an average debt of 31.5% of GDP. This new target refers to the average debt in the same sample of countries in the same period but considering public and private external debt[16]. I limit the coefficient of relative risk aversion to 10, in accordance with the discussion of the previous section. Such restriction leads to a ratio between volatilities of consumption and GDP of 1.05 instead of 0.98, but the other three targeted statistics are met. The average holding of international reserves declines to 5.4% of GDP, still indicating that the optimal policy is to accumulate assets and liabilities simultaneously[17].

In order to provide assess the role of rigidities in the government budget constraint, I solve the model changing equations (3) and (4) to (13) and (14) respectively. The insertion of this fixed government expenditure makes the adjustment to adverse shocks costlier and improves the quantitative performance of the model. When Bianchi, Hatchondo and Martinez (2018) recalibrate their model with $g = 0$, instead of $g = 0.12$, the average level of reserves falls from 6% to 3%. I insert the fixed government expenditure in my benchmark model with the same value of $g = 0.12$. Results appear in column 3 of Table 3. The average level of reserves increases from 7.7% to 8.9% and other statistics, targeted or not, do not change meaningfully. Such change indicates that fiscal rigidities also play a role in an economy with short-term debt.

$$c_t + g = y_t + q_t d_{t+1} - d_t - q^a a_{t+1} + a_t \tag{13}$$

$$c_t + g = y_t^a - q_t^a a_{t+1} + a_t \tag{14}$$

The remaining columns in Table 3 show robustness checks for the value of the coefficient of risk aversion. Changing it to $\sigma = 3.3$, as Bianchi, Hatchondo and Martinez (2018), and recalibrating the other parameters ($\beta = 0.92$, $\psi = 0.87$, and $\kappa = 5$), the model delivers similar results, with the stock of reserves declining from 7.7% to 5.7% of GDP. Reducing the coefficient of relative risk aversion to $\sigma = 2$, columns 5 and 6, leads to excessive consumption volatility, even with a new choice of parameters to meet the same targets ($\beta = 0.92, \psi = 0.88$, and $\kappa = 3$). In both cases, the mean level of reserves falls to

[16] information from the updated and extended dataset of Lane and Milesi-Ferreti (2007).
[17] If I restrict $\sigma = 5$, the model, recalibrated to meet the same targets, produces mean reserves of 3.5% of GDP.

approximately 2% of GDP[18]. To such a degree, the optimal policy still is to hold positive amounts of international reserves.

5. CONCLUSION

I show that the combination of three facts currently observed in emerging markets – i) high level of international reserves, ii) positive amount of sovereign debt, and iii) positive interest rate spread – is compatible with results from a quantitative model of sovereign default in which these variables are endogenous. In this structure, the only use of reserves is to smooth consumption, even after a default, when the economy is excluded from international financial markets.

Differently from previous studies, I focus on the roles of partial default to generate the above-mentioned trio. In this case, the joint accumulation of assets and liabilities does not erode debt sustainability as much as under full debt repudiation. While a higher stock of foreign exchange reserves increases the value of defaulting, higher debt decreases it. The last effect occurs owing to governments carrying a share of their previous liabilities upon reentry on international debt markets after a default. In this setting, governments accumulate debt and reserves during periods of economic growth and deplete the former as the boom fades away. This leads to reserves being positively correlated with debt and output and negatively with spreads, in accordance with the data for emerging markets in the last decade. The addition of risk-averse lenders in the model increases the optimal level of international reserves due to an amplification of the precautionary motive. With this feature, spreads rise even more during bad times than under risk-neutral pricing, because both the default risk and the risk premium increase.

[18] Hernandez (2016) is the only other paper in this framework to obtain positive amounts of both debt (15.9%) and reserves (4.0%) while also presenting sensible average interest rate spreads (180 bps) using $\sigma = 2$. However, his calibration of the endowment process is more than twice more volatile than suggested by Uribe and Schimitt-Grohé (2017) for quarterly frequency data. He obtains it based on the Mexican GDP multiplied by its real exchange rate. His defense of this choice relates to differences of the exchange rates regimes in Mexico and Argentina, the most frequent example in models of quantitative sovereign default. Volatile endowment processes help to achieve a solution with positive reserve accumulation using a lower coefficient of risk aversion. A high calibration of the volatility of income also appears in Alfaro and Kanczuk (2017).

The model has a good quantitative performance and suggests that the optimal policy is to hold a positive quantity of foreign exchange reserves. Nonetheless, it does not reproduce the total volume of assets held by emerging countries' governments in the last decade. I consider that the present model offers a starting point for the discussion on the optimal level of international reserves, since there are other reasons to hold them beyond consumption smoothing – as indicated by Gosh et al (2016), Obstfeld, Shambaugh, and Taylor (2010), and Aizenman and Lee (2007).

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